Adaptive search approach in the multidisciplinary optimization of lightweight structures using hybrid metaheuristics

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Abstract

Within the last few years environmental regulations, safety requirements and market competitions forced the automotive industry to open up a wide range of new technologies. Lightweight design is considered as one of the most innovative concepts to fulfil environmental, safety and many other objectives at competitive prices. Choosing the best design and production process in the development period is the most significant link in the automobile production chain. A wide range of design and process parameters needs to be evaluated to achieve numerous goals of production. These goals often stand in conflict with each other. In addition to the variation of the concepts and following the objectives, some limitations such as manufacturing restrictions, financial limits, and deadlines influence the choice of the best combination of variables. Thus, an accurate, reliable, and fast optimization tool would be necessary for a continuous development process between engineering departments.

This study introduces a structural optimization tool for assemblies made of sheet metal, e.g. the automobile body, based on parametrization and evaluation of concepts in CAD and CAE. This methodology focuses on those concepts, which leads to the use of the right amount of light and strong material in the right place, instead of substituting the whole structure with the new material. To consider the manufacturing limitations of sheet parts and additional costs for multiple-piece structures, a constraint model is proposed which is embedded in the optimization tool.

An adaptive hybrid metaheuristic algorithm is designed to eliminate all factors that would lead to a local minimum instead of global optimum. Finding the global optimum is granted by using some explorative and exploitative search heuristics, which are intelligently organized by a central controller. Reliability, accuracy and the speed of the proposed algorithm are validated via a comparative study with similar algorithms for an academic optimization problem, which shows valuable results.

Since structures might be subject to a wide range of load cases, e.g. static, cyclic, dynamic, temperature-dependent etc., these requirements need to be addressed by a multidisciplinary optimization algorithm. To handle the nonlinear response of objectives and to tackle the time-consuming FEM analyses in crash situations, a surrogate model is implemented in the optimization tool. The ability of such tool to present the optimum results in multi-objective problems is improved by using some user-selected fitness functions.

Finally, an exemplary sub-assembly made of sheet metal parts from a car body is optimized to enhance both, static load case and crashworthiness. It is shown that the desired objectives under cost and manufacturing limitations could simply be obtained from predefined concepts using a simulation-based optimization tool.

Keywords: Structural optimization, Optimization algorithms, Lightweight design, Crashworthiness, Multi-Objective, Manufacturing limitations, Metaheuristics
Kurzfassung


In dieser Studie werden strukturelle Optimierungswerkzeuge für aus Blech gefertigte Baugruppen, z. Karosserie, basierend auf Parametrisierung und Bewertung von Lösungen in CAD bzw. CAE. Diese Methodik konzentriert sich auf die Lösungen, die dazu führen, dass die richtige Menge an leichtem / festem Material an der richtigen Stelle der Struktur verwendet wird, anstatt vollständig ersetzt zu werden. Um die Einschränkungen bei der Herstellung von Blechteilen und die zusätzlichen Kosten für Strukturen mit mehreren Teilen zu berücksichtigen, wird ein Integritätsmodell vorgeschlagen, das in die Optimierungswerkzeuge eingebettet ist.


Eine Unterbaugruppe aus Blechteilen, die zur Automobilkarosserie gehören, ist optimiert, um beide zu verbessern; statischer Lastfall und Crashtest sicherheit. Es wird gezeigt, dass gewünschte Ziele unter Kosten- und Herstellungsbeschränkungen einfach aus vordefinierten Lösungen mit simulationsbasierten Optimierungswerkzeugen erhalten werden können.

Schlüsselwörter: Strukturoptimierung, Optimierungsalgorithmus, Leichtbau, Crashworthiness, Multi-Objektiv, Fertigungsbeschränkungen, Metaheuristik
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1. Introduction

Requirements generate concepts and concepts are directed by limitations. Eventually, a methodology is needed to find the best concept to meet the requirements with respect to the limitations. If wide ranges of concepts are available and if the satisfaction of objectives for each concept is measurable, the only remaining task is to find the concept that simultaneously fulfills all objectives and acts within the limitation level. This study employs above-mentioned principles to design an optimization tool to improve crashworthiness of sheet metal structures in the automotive industry.

1.1 Motivation

Transport industry consumes about 30 percent of energy in Europa and is a significant producer of greenhouse gas emissions [190]. Automobiles produce the largest shares of emissions. For this reason, automotive industry aims at finding applicable solutions to reduce the effects of environmental damage. Increasing the engines’ efficiency to reduce fuel consumption, designing more aerodynamic bodies, reducing the energy consumption of electrical components, developing hybrid cars and finally making the bodies lighter are considered as the most significant methods to reduce the emissions. Recent observations show that reducing 100 kg of weight caused the reduction of 8.5 gram CO$_2$/km [191]. Based on the current European directives the produced CO$_2$ in each 100 km should not exceed 130 gram per 100 km. This value will reduce to 95 gram in 2020 [192]. Weight reduction of the automobile body is considered as the most relevant solution to fulfill environmental regulations.

However, reducing CO$_2$ emissions is not the single aim of the automotive industry. Another motivation, which confronts this industry with a big challenge, is the strictness of safety regulations that took place during recent years. In this field, many active solutions try to prevent accidents. However, in an accident, the automobile body, airbags, and seat restraint systems play an important role to ensure the survival of the occupants. Stiffening the structure reduces the intrusion of parts into the compartment, while it decreases the transferred deceleration peak to the occupants.

In addition to safety regulations, some functional requirements, like bending, the torsional stiffness of the body and endurance strengths, push the designers to think about stronger structures. Lighter and stronger structures are not enough because they should be achieved with a specific predefined budget. Nevertheless, producers of expensive products would disappear from the market in a short time.

During the development process, designer must also consider different kinds of manufacturing restrictions. Otherwise, the tooling designers will reject non-producible parts.
As a brief conclusion, designing lighter and stronger structures under cost and manufacturing limitations are considered as the huge challenges of automakers. Roughly, two well-known solutions have been employed to simultaneously satisfy these targets. One is the modification of cross-sections and using ribs and stiffeners, the second is the substitution of conventional materials like steel with lighter materials like polymers and composites. Because of the production limitations, the final cost, and the maximum expected strength, direct replacement of lighter parts without considering to specific conditions and restrictions is not easily applicable.

In most cases, it will be necessary to replace a certain percentage of original parts with new materials. By this way, a combination of two or more parts will be generated instead of one unique material. Through this process, the geometry of original parts might be changed as well. In recent years, these challenges taken into account in order to create applicable strategies to generate lighter structures by using different materials.

Different kind of concepts and their combination creates a wide range of input data, which are called input variables. There are three types of variables, i.e. continuous like geometrical dimensions and discrete variables like material properties and mixed variables. A huge number of variables offer a large variety of concepts and are considered as an advantage. However, it leads to create a massive design space which increases the optimization time.

On the other hand, in structural optimization, more than one objective is envisaged. To consider all objectives, different methods have been suggested with some advantages and disadvantages. Choosing the inappropriate strategy leads to infeasible results and financial loss for the producer.

Also the limitations need to be handled in an optimization process in such a way, that not only eliminate the severe infeasible concepts, but keep the good concepts with less infeasibility in optimization loops. In addition to generating and evaluating of initial concepts, structural optimization tool need to find the optimum concept for the designer in the end. This can be achieved by optimization algorithms. When objective is able to be written as a function related to input variables, gradient-based methods will be used to find the optimum concept. Otherwise, by using the DOE method, a portion of design space is selected and will be evaluated until the best solution can be found. When more accuracy is required, an approximated model of objective could be constructed to estimate the behavior of objective. If design space is extremely huge with discrete variables and the structure shows nonlinear behaviors the above-mentioned methods become faulty and expensive. In such a case, metaheuristic algorithms are able to find the optimum result by iteratively enhancing the good concepts. However, metaheuristics are considered as expensive algorithms because of their
slow convergence rate. They should be improved or combined with other heuristics to overcome their deficiencies.

1.2 Aim of the study

This study tries to introduce a comprehensive tool for structural optimization considering the above-mentioned criteria and independent on problem types. This tool primarily able to realize concepts in CAD. Then it proposes a hybrid metaheuristic to find the optimum structure from evaluated concepts. It should be implemented without any restrictions of type and the number of variables, objectives, and limitations. On the other hand, it has to work with any type and number of load cases. In optimization algorithms, every probability, which may lead algorithms towards convergence in local optima, must be intelligently restricted. The proposed optimization tool tries to use general programing languages and simple logics to generate initial concepts in CAD and fluent interaction of CAD and CAE software on industrial level. The proposed tool has been utilized with a surrogate model to reduce the optimization time in case of dynamic loads and noisy responses.

1.3 Structure of the study

In order to better describe the backgrounds of the proposed optimization tool of this study, strong and weak points of similar research are investigated in the field of structural optimization. In parallel, complexities and challenges of structural optimization will be determined and discussed. Hence, the second chapter presents a comprehensive survey of different types of structural optimization methods, which have been evolved and categorized during the years. Since, in this research, sheet structure are the main goal of optimization, topology and shape optimization will be shortly discussed. However, optimization of size, material combination, and part types are debated in more detail. During the classification of optimization types, the necessity of efficient algorithms starts to show itself in structural optimization. Therefore, various types of optimization algorithms are presented, which are more applicable in field of optimization in mechanical structures. Since, this study going to use hybrid metaheuristics for its optimization tool, these groups of algorithms will be thoroughly investigated. In the following, four controversial groups of complexities are introduced which play significant roles in the designation of optimization tool.

As explained in the introduction, optimization algorithms are one of three milestones of optimization tools. Hence, chapter three collects an archive of strong and weak points of some applicable algorithms in the field of structural optimization. Based on several evidences from experimental research, a combination of some strong features of algorithms could find the global optimum with more reliability. It will be shown that to achieve this reliability, an intelligent organization of algorithms during the optimization loops is required. Therefore, a central
processor is designed and employed to control the overall functionality of hybrid algorithms. This processor coordinates the internal operators and communication of algorithms using several control parameters. At the end of chapter three, the proposed adaptive hybrid algorithm is employed to optimize a well-known academic optimization problem, which introduces better accuracy and quickness compared to similar research from literature.

Chapter four presents the proposed optimization tools by optimizing a generic sheet metal structure, i.e. a B-pillar using a lightweight design approach. The reason for choosing a B-pillar is the existence of different load cases with a wide range of proposed solutions. They challenge the optimization process with multidisciplinary and mixed-variable optimization. It will be shown how three principal parts of the optimization tool realize and evaluate the concepts and how the optimization algorithm finds the optimum concept. In order to consider manufacturing restrictions, a penalty scheme reduces the chance of structures that are difficult to manufacture to participate in optimization loops. To calculate the extra production cost for multiple-piece structures a cost model is embedded in the optimization tool and gives more chance to less-piece structures. In addition, a surrogate model will be presented in chapter four to estimate the nonlinear behavior of structures under crash loads. At the end of the chapter, a comparative study on optimization results using different combinations of objectives is introduced. Through these combinations, designer will be able to see and select some optimums near to a global optimum with different solution architectures.
2. State of the art in structural optimization

2.1 Classification of structural optimization types

Evaluation and investigation of academic research, commercial software, and industrial projects show that manufacturing processes play more significant roles in the classification of structural optimization types than others like algorithms. Before classifying of structural optimization based on manufacturing techniques, a survey of other types of taxonomies and related criteria is presented, which has been considered by researchers during recent years. In this classification, three types of references are considered:

1- Scientific publications
2- PhD-theses
3- Commercial software

Since every above-mentioned source has different horizons in the classification of structural optimization, a tailored combination of their advantages may introduce a taxonomy that is more practical for implementation.

Structural optimizations types in scientific publications

In this section, recent publications dealing with structural optimization are investigated, which have classified the types of structural optimizations and optimization methods in general. Schumacher [1] proposes five forms of structural optimization based on design variables:

![Classification of optimization problems based on design variables, according to Schumacher [1].](image)
- Dimensioning: wall thickness, dimension of cross-sections, and stacking sequences
- Form optimization: leads to the optimization of the outer boundaries of parts
- Topology optimization: deals with elements of parts not dimensions or forms
- Material selection: finds the optimum material for parts
- Part type selection: essentially deals with the kind of parts, not their dimensions

In his study, Schumacher also presents the main topics of structural optimization, which are discussed in next the sections.

Garret [2] also focuses on optimizations methods. What kind of algorithms should be used for different types of structures and their variables and constraints are evaluated in his research as well. He divides structural optimization into four groups and addresses four solution methods:

- FEM
- Sensitivity analysis
- Approximation techniques
- Topology optimization

Garret also presents some industrial examples and two main groups of structure or optimization approaches are mentioned. The first method eliminates unnecessary regions of parts, which have less influence on strength, or any other kind of performance. In this method, there is no need to define dimensional parameters for parts and it deals with a 3D design space. The second method deal with such parts, which are define by specific dimensions as thickness, angles, and diameters, and in contrast to the first group, no design space is considered here.

Yang [4] mentions that there is a lack of a clear classification for structural optimization in literature and especially some confusion in the suitable words in this field of study. He classifies optimization methods as shown in Figure 2.2:
Rao [6] introduces the same categorization as Yang [4] and presents eight different groups.

**Figure 2.2.** Classification of optimization problems, presented by Yang [4].

**Figure 2.3.** Classification of optimization problems, presented by Rao [6].
In “nature of the equation”, there are some cases, which the fitness function is expressed as an equation consist of input variables. In order to have an efficient solution, either linear and nonlinear, geometric programing or quadratic programing should be utilized. Haung [7] divides the structural optimization into three groups: size, shape, and topology. He mentions that the topology optimization method could be replaced by simply restricting the structural modification to the existing boundaries.

The following authors have considered the optimization method in general (not especially structural optimization). Their studies will be more relevant to the next section of this study when optimization algorithms and the challenges of optimization methods will be presented.


Georgios [11] has identified three groups of structural optimization from recent publications. First group; optimize the material properties [78, 79, 252]. Second group; try to minimize the stress concentration [76]. Third group: find the optimum dimension of cross-sections through a profile [75]. He applied a shape optimization problem in his study.

Saitou [15] explains in his survey about structural optimization:

“Structural optimization is a class of optimization problem where the evolution of an objective function (s) or constrains requires the use of structural analysis (typically FEM)”. Eschenauer [141] divided optimization problems into a three-column-concept in 1988, which introduces all possible scenarios to obtain optimal design in structural optimization.
Until now, different types of structural optimization from researchers’ point of view in scientific publications and PhD-theses have been assessed. Since providers of commercial software already move in the direction that support the industrial requirements, assessing optimization software could be useful to realize another classification type. In the following, some well-known structural optimization software is introduced.

**Classification of structural optimization types in PhD-theses**

Stoffel [8] in his PhD-thesis recognized three types of structural optimization:

"Based on the theory of PDE-constraint optimization, descent methods in structural optimization require knowledge of the (partial) derivatives with respect to shape or topology variations. Therefore, shape and topology sensitivity analysis is introduced and the Topological-Shape Sensitivity Method gives the connection between both sensitivities."
method leads to a systematic procedure to compute the topological derivative by terms of the shape sensitivity”.

Giger [9] determined two groups for the optimization of structures as shape/size and topology optimization. He implemented a size optimization in his thesis using his adaptive evolutionary algorithm.

Wehrle [10], in addition to three types of structural optimization, has extended material optimization to topology, shape and size optimization. To overcome the discrete behavior due to material selection, he introduced a continualization approach for design variables [13].

Fiebig [86] considered two different criteria for the classification of optimization problems within the structure. The first group of criteria for describing general optimization problems defines the mathematical framework. The second group summarizes features of tasks within the structure optimization. Here, more details with regard to the properties of every criteria and length of design variable vector is illustrated:

He explained that the properties of the criteria could be combined with one another, so that, for example, binary parameters with 10,000 dimensions can be multi-criteria without constraints in a linear optimization.
Structural optimization types in commercial software

TOSCA structure [16] provides three groups of topology, size/shape, and bead optimization for solid and shell parts. TOSCA improves the static and dynamic behavior of shell parts without changing the thickness or connection state with other parts.

Vanderplaats R&D [17] has introduced Genesis for structural optimization, using three groups: 1) grid coordinates, 2) geometry properties, 3) material properties. Following this tool, the user is able to optimize five different types of structural varieties: shape, topography, size, topometry, and topology. The topometry module gives the opportunity of altering the thickness to obtain stronger sections.

MSC Nastan [18] enables the user to implement two or more optimization types at the same time. That means size, form, topometry, topography, and topology may combine into a single optimization task.

LS-DYNA [19] has also provided a wide range of optimization methods and tools for the main groups: size, shape, and topology.

OptiStruct [20], in addition to three main groups, has introduced individual modules for composite laminate and additive manufacturing optimization.

As a short conclusion, different types of optimization, received here from resource studies and commercial software, are classified as topology optimization and material/size/shape optimization.

2.1.1 Topology optimization

Since topology optimization is not the main goal of this study, only the basic definitions are introduced here. Topology optimization provides an essential and basic answer to designers how to set out the material in predefined design space to obtain the optimum performance.

The initial idea of topology optimization was designed for mechanical structures in the first place, but extended for acoustics, electromagnetics, liquids, etc. later. Sigmund [21] accomplished a comprehensive survey of different types of topology optimization in “Topology Optimization Approaches”.

Density approach: simplified isotropic material with penalization with the aim of reducing the complexities of the initial model of topology optimization proposed by Bendsøe and Sigmund. Later on, these researchers themselves physically enhanced the SIMP method. In order to introduce restriction on density variables, some other methods were presented in studies [21].

Topological derivation: Topological derivation in shape/topology optimization is based on the prediction of influence of infinitesimal holes in every place of design space, which use the results to make the new hole [298].
**Level set approach:** In this approach, the zero level contour of the level set function defines the boundary of the design and the structure is defined by the domain where the level set function takes positive values.

**Phase filed approach:** This method works directly on the density variables and considers minimization of the functional consist of a double well function with internal parameters like interfacial thickness and weight factor.

**Discrete approach:** As topology optimization uses discrete variables, it seems normal that it uses discrete methods to solve problems. However, many kind of discrete topology optimization could be considered as a continuous problem.

**Lagrange approach and combined shape and topology optimization**

Schumacher [298] presented a simultaneous method of shape and topology optimization to iteratively position a new hole into the structure.

There are some new trends of applications in topology optimization, e.g. cost and weight optimization of hybrid parts using a multi-material topology optimization approach [270]. An extended cost calculation model was used to estimate the manufacturing costs of the hybrid component based on its geometrical properties.

However, topology optimization involves some challenges for the usage in automotive lightweight design technologies [297]. “First, the technical aspects like crashworthiness and acoustic requirements should be implemented into the topology optimization. The second future path focuses on sheet structures and hybrid parts. The third challenge is the integration of manufacturing simulation. The last future path treats a continuous and integrated development process”.

### 2.1.2 Size optimization

In this section, size optimization is introduced from researchers’ points of view in scientific papers and most common algorithms for size optimization will be presented in the next section. Parts made of sheet material with unique thickness are the largest share of automobile bodies. Stiffness of these parts is constant over the surface unless a form deviation happens [189, 299]. It is already possible to change the form of different regions of sheet parts to prepare them for more load support. Usually, these two processes, design and development, are sequentially executed in industry. In that way, designers presents the first concept of structure, which covers some functional requirements of components. Later, experienced experts from manufacturing or test departments try to evaluate all performances of the structure. These evaluations could be executed virtually or physically, but lead to some modifications of the
initial design. These modifications should be implemented inside the design boundary as predefined by the designer. Final improvements of geometries and material properties of the initial structure to obtain better performance are simply called structural optimization.

During recent years, automotive industry has tried to reduce the overall weight of car bodies using stronger and lighter materials besides modification of the parts’ dimensions [189, 194]. Dimensional modification in CAD software and evaluation of new designs in CAE software have improved in parallel with the development of these software and their interfaces.

The necessity of good and accurate models for structural optimization increases when the number of variables rise due to variation of concepts. The results of size optimization have been influenced during the years by new technologies. In fact, an applicable optimization tool should be able to consider the effects of every new concept without any limitations in the current optimization process. For instance, Figure 2.5 left shows the connection of two parts in assembly before introducing laser welding and Figure 2.5 right shows the improved form of assembly using laser welding process. This process has lead to a weight reduction of 50%.

**Figure 2.5.** Effect of new technologies on result of optimization. The format of contact between welded parts has been entirely changed because of using laser welding (right) instead of traditional arc welding process (left)

In the following, a brief overview on methods and strategies in the size optimization of mechanical structures is presented. Regardless of the type of optimization algorithms, researchers have been interested in finding the best dimensions of a sheet structure via two groups [199, 253, and 299]. The first group is parameter-based that generates different geometries from initial design, for example by changing the length or angle of a line from the section profile [242, 244]. Using CAD capabilities allows complex and logical geometries to be generated and evaluated [19, 300, 301, and 302]. The second group generates different geometries for a profile by changing nodes located on a line or curve without having any parametric geometry [198]. These methods are called free-parameters and involve a large scale of the design variables. There is a need for a strong mechanism to prevent the zigzag phenomenon [199]. Due to the mentioned weaknesses compared to the parameter-based method, the free-parameter method has been shown less of attention and investigation by researchers.
2.1.3 Optimization of material combination

As mentioned in section 2.1.2, altering of part materials in structures, besides the modification of cross-section sizes, takes into account two well-known approaches to reduce the weight and increase the strength of structures in automotive industry. In this section, the circumstances of choosing the proper material for cross-sections is evaluated. This process, which is called “material selection” or “material management”, expected to replace stronger materials in places that need more strength to bear the applied loads [84]. That means traditional materials are replaced by lighter and stronger materials in the part or structure. Since these kind of materials are relatively expensive compared to the traditional ones, it is desirable to use only a minimum amount of them [194]. Recent studies show that optimization of structures would not be efficient only by optimizing the material combination and it should go along with size optimization. Barchini has designed a method, which is able to select the proper material and geometry simultaneously with the aim of weight reduction. His method is based on generating an individual library for material and geometry. He has carried out a combination of a genetic algorithm (GA) and a backtracking algorithm to find the optimum geometry and material [303]. Poulikidau and his colleagues have introduced a material selection method to replace materials in a structure with the aim of weight and environmental impact reduction simultaneously [304]. Graccobi in a multi-material design process has improved the filtering operator to overcome the huge variety of selections [305]. Ermolaeva has also implemented a comprehensive study on material selection of an automotive structure with integration of structural optimization and environmental impact [306]. Aly has introduced a method for material selection in sandwich beams through parameter optimization [307]. In addition, Ashby and colleagues have explored ways of designing hybrid materials, emphasizing the choice of components, their shape and their scale [308]. Singh has used the novel feature to integrate shape and material to model and visualize multi-objective selection problems [309].

In the next sections, it will be shown that simultaneous material and size optimization slow down the optimization process because of the longer variable vector. In chapter 2.3, by evaluating recent studies on structural optimization methods, some well-known approaches to overcome this complexity will be introduced. In chapter 3, the proposed methodology of this study to find the optimum size and material combination in a multi-material structure will be introduced.
2.1.4 Part type optimization

This section discusses another type of structural optimization that is not part of the mentioned classes of chapters 2.1.1, 2.1.2, and 2.1.3. With a short review on the priority of concept implementation in structural optimization, the necessity of this approach will be shown. When size modification, thickness adjustment, and material replacement do not satisfy the design requirements, another method, which is called part type optimization [1, 42], could be the last solution. This method of structural improvement is implemented by adding a new shape of assembly parts to industry. As an example, using a filler between two assembly parts leads to an increased bending stiffness and is considered as a common solution in this category [100, 283]. These filler materials could be made of cellular parts, ribs, or of same materials as the base parts.

Innovation in manufacturing and assembly technologies already brings better and feasible ideas to reduce the complexities of productions and cost reduction. Therefore, a comprehensive structural optimization method should be able to accept or consider any kind of new parts in its concept pool. As an example, currently, polymer ribs and metal foams are added to metal structures in an individual process, which leads to weak interaction between dissimilar materials. In future, the possibility of producing dissimilar material in a single process could be considered as a simple and applicable method in structural optimization in series production lines.
2.1.5 Shape optimization

Similar to size optimization, shape optimization enhances the existing structure for applied loads, with the difference that shape optimization is usually used for solid parts instead of shell parts.

This fine-tuning consists of the modification of the outer skin of solid parts and movement of internal nodes. That means that the part thickness will change in some sections at the end, see Figure 2.6. Normally, shape optimization tries to eliminate stress concentration around holes and sharp edges by changing the form or curve of mentioned surfaces. The curve modification technique must be carefully implemented because it could easily destroy the mesh properties and reduce the strength of related places [181]. Haftka [22] proposed three well-known techniques of boundary modification and the elimination of stress concentration.

1. Polynomial representation of boundaries

Thickness distribution might be defined by using polynomials. Thereby, designers will able to change the thickness of different sections by changing the curve coefficients. Bharikatti [23] defined a third order curve for edges of a power transmission rod under torsional load. He changed four design variables to control the shape of the design curve to minimize the parts’ volume in respect to stress concentration constraint. Hsu [24], in a review of shape optimization methods, introduced a comprehensive flowchart of shape optimization process, Figure 2.7.
He divided the optimization algorithms into two main groups: sequential approximation and direct search approach. Ha also compared and discussed the output results of two well-known examples in literature: the tension bar and the torque arm. Huan [310] used shape optimization to find the optimum dimension of cross-sections of the body in white. His objectives were weight reduction, increasing of static strength, and Eigen frequency with regard to manufacturing costs. He used an improved genetic algorithm (GA) to find the optimum design variables.

2. Spline representative boundaries
Haftka [22] mentioned an oscillatory boundary shape as a weakness of the polynomial method and proposed spline to obtain maximum smoothness.

3. The design element concept
In this approach, the whole part is divided into small elements and some key nodes control the elements. Thus, the form of different regions could be modified using a new coordination. Rozvany [137] discussed FE-based generalized shape optimization. He classified them with respect to the types of topologies involved, namely isotropic-solid/empty, anisotropic-solid/empty, and isotropic-solid/empty/porous topologies. He reviewed the origins, theoretical background, history, range of validity, and major advantages of the SIMP (Solid Isotropic Microstructure with Penalization) method, which was initially introduced by Bendose [73].
2.2 Classification of optimization algorithms

In the previous chapter, structural optimization and its characteristics were presented. This chapter introduces and classifies optimization algorithms. Since all optimization algorithms are not directly involved with structural optimization, here more relevant algorithms and approaches will be investigated. Other, less relevant and seldom used algorithms are only introduced by name.

Schumacher [1] introduced seven types of algorithms from literature for structural optimization:

1- Optimization with constraints
2- Optimization without constraints
3- Approximation of real problems
4- Approximation-based optimization algorithms
5- Stochastic search strategies
6- Solving for discrete optimization
7- Combination methods

Garret [2] introduced a number of optimization algorithms as follow:

1- Unconstrained function of variables
2- Constrained function of N variables: linear programing
3- Constrained function of N variables: sequential unconstrained, minimization techniques
4- Constrained function of N variables: direct methods
5- Approximation techniques

Sokolowski [97], Arora [25], Kacprzyk [55], Kirsch [72], Adeli [132], Harzheim [136], Haftka [142] and Zelinka [138] have presented almost the same optimization algorithms.

Spillers [98] presented various types of optimization methods as tools of optimization. Most of the presented methods by Spiller are mathematic-based expect for the eighth group which is named evolutionary algorithms.

Yang [4] divided the optimization techniques into two groups, consisting of deterministic and stochastic methods. The first group follows a specific procedure and all variables and functions are repeatable like the Hill climbing algorithm. In contrast, stochastic methods involve randomized processes like GA.
Rao [6] proposed eight methods for optimization:

1- Linear programing
2- Non-Linear programing
3- Geometric programing
4- Dynamic programing
5- Integer programing
6- Stochastic programing
7- Optimality criteria
8- Modern methods

Ghiasi [26] classified two main groups for the special case of stacking sequence optimization:

1- Gradient-based
2- Direct search

More or less, a similar classification of optimization methods is found in commercial software:

LS-Opt [19] uses two groups of algorithms to optimize structural optimization problems:

1- Successive response surface in nonlinear problems, i.e. collisions and metal forming
2- GA for ordinary problems with lots of local minima.

The ESTECO [311] software group has two groups of optimization algorithms:

1- RSM-based
2- Direct optimization

It also suggests that if users need more reliability for global optimum, they need to use heuristics consisting of simulated annealing [90], GA [228], and PSO [296]. In contrast, when it needs more speed to obtain optimum results it proposes gradient-based methods, like classical SQP, and Bounde BFGS methods [311]. Hybrid and simplex algorithms were suggested for general conditions. ESTECO provides the ability of assessing the relation of
input variables and objectives before the optimization process, using basic post-processing. It will help to recognize variables that are more important and to increase the optimization speed in coming iterations.

SHERPA [27] proposes two group of optimization algorithms (introduced by Heed’s company)

1- Local methods like Levenberg-Marquardt and Simplex
2- Global methods like Monte Carlo and Grid search

SHERPA states that using a unique search strategy, it is capable hybrid exploration with quite reliable results and an adaptive setting. During the multi-dimensional search with some methods, SHERPA takes the advantages of each method and reduces the contribution of ineffective approaches in the next steps to increase the overall performance of algorithms. The robustness and efficiency of SHERPA has been compared with GA, SA, NLSQP, and RSM to optimize some academics problem [27].

Isight [28, 148], an optimization module of Dassault Systems, introduces a wide range of optimization algorithms. It provides a control panel for the user to choose type of algorithm, solver type, and status of illustration and choose save formats.

Optimus [57] introduces a wide panel of FEM software in an engineering optimization module. The user is able to add three group of optimization algorithms to optimize mechanical structures under different load cases.

1- Local optimization method

NLPQL, Sequential Quadratic Progressing (SQP), Generalized Reduced Gradient and some similar methods are useful to solve constrained problems and normally they are quickly converged to a local minimum. They need to define objective functions, constraints, and input variables to make the sensitivity-analysis.

2- Global optimization method

Superior abilities to find global optimum using adapted evolutionary algorithms

3- Adaptive hybrid method

Two methods, adaptive region optimization and efficient global optimization help to find the global optimum after response surface search (make RSM finer).

Choi [103] compared the design capability of three optimization software, i.e. MSC Nastran, Genesis, and OptiStruct in terms of shape, topology, topometry, and topography optimization.
Up to this point, different methods of structural optimization algorithms have been introduced from literature and commercial software. As a conclusion of these evaluations, three methods of optimization algorithms could be classified in structural application as follows.

2.2.1 Approximation techniques

The approximation method in structural optimization consists of all techniques and approaches which select an area of the whole design space, implement FEM, and connect design variables to FEM results with a surrogate model [158]. In big design spaces and extensive nonlinear problems huge portions of samples need to be selected to guarantee the accuracy of the surrogate model. A few methods have been developed to obtain a surrogate model.

1. Polynomial regression

The response surface method is implemented to obtain an estimation of response functions in terms of design variables. This model is normally written as mathematical definition.

\[
y = F(x_1, x_2, ..., x_n) + \varepsilon
\]

Where \( y \) is response and \( x_i (i = 0, 1, ..., n) \) is design variable and \( \varepsilon \) is error. Function \( F \) is normally selected in polynomial form as follows:

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2
\]

Where \( \beta_i (i = 0, 1, ..., 5) \) is obtained from matrix relation with known \( y \) and \( x \).

Myers [29] introduced a comprehensive description of the response surface methodology and approximation approaches. In highly nonlinear problems the accuracy of linear or quadratic polynomial regression may not be sufficiently exact. One method of increasing the accuracy of this model is using a higher order in the regression model. Other approaches to obtain more accuracy is using more design samples in model construction. If evaluation of samples are not time-consuming, then using more samples to make accurate models is considered as the better solution. Hofwing [266] studied in which cases increasing some potential high-order term is enough to have an accurate polynomial. He employed genetic algorithm to find the optimal polynomial regression model and examined his model on Rosenbrock function to obtain constant coefficients. Comparing the results of the OPRM (optimal polynomial regression model) with the quadratic regression model with RMSE (Root mean square error) shows better accuracy in his proposed model. Jonsson [30] has focused RSM and space mapping techniques to be strong tolls in regard crashworthiness and metal forming optimization. He compared his model performance with traditional RSM by four different examples. Todoroki [31] employed RSM for a stacking sequence optimization of composite structures under buckling load.
2. Moving least square

A nonlinear least square problem is an unconstrained optimization problem of the form:

$$\min F(x) = \frac{1}{2} \sum_{i=1}^{m} r_i(x)^2$$

where $$n$$ is the number of variables and objective function $$F(x)$$ is defined by $$m$$ axillary residual functions $$\{r_i(x)\}$$ with assumption of $$m \geq n$$. Since the minimization of the sum of squares of residual function is the aim of this method, it is called least square method.

Song [32] has used the moving least square method in structural optimization of knuckle under bump and brake loads. Zadeh [33] employed the moving least square method to obtain the optimum value of the second moment of area, depth of beam, and fiber volume fraction to reduce the weight of the cantilever beam made of composite laminate.

3. Kriging

The kriging model estimates a combination of polynomial models plus leaving of the form given by $$Y(x) = F(x) - Z(x)$$ where $$Y(x)$$ is the unknown function of interest, $$F(x)$$ is the known polynomial function of $$x$$ and $$Z$$ is the realization of a normally distributed Gaussian random process with mean zero variance $$\sigma^2$$ and non-zero covariance [34]. Gaspar [35] used the kriging method to predict the failure probability of a structure, which needs high time-consuming FEM and obtains a relatively good performance compared to other traditional methods. Skata [36] implemented the Kriging method in the optimization of stiffened cylinders for an Eigen frequency problem. He compared ability and performance of the Kriging method with the neural network method. Koch [37] employed the Kriging to optimize a multidisciplinary problem in an oil tanker ship. He considered three criteria, reliability, robustness, and signal level, to measure the design quality or optimization.

4. Neural network

Neural networks introduce as an effective tools for function approximation. Multilayer perceptron with one input layer, two hidden layers and one output neuron is commonly employed to approximate the fitness.

5. Fuzzy logic

Many optimization problem are based on the acquiring of accurate mathematical terms. On the other hand, in some real problems, the input variables, objectives, and restrictions may found as indistinct and linguistic terms. In such cases, fuzzy theories can be employed to model or estimate the fitness function.

6. Radial basic function
This approach is employed to estimate function of scattered multivariate data. To approximate response functions both linear combinations of radially symmetric function based on Euclidean distance or other such metric are used. It generates good fitness to arbitrary contours not only for deterministic but also for stochastic response functions [166].

2.2.2 Direct methods, evolutionary algorithms, and metaheuristics

These methods do not use any estimation in order to find the best combinations, but find iteratively the best combination of input variables [88, 139]. These algorithms only differ in the way they explore around the best combinations. Ali [180] presented a survey of metaheuristics with an excellent classification, Figure 2.8.

**Figure 2.8.** The classification of metaheuristics includes single and population-based [180]

Scatter search makes new trial solutions by combining reference solutions and using strategic designs that exploit knowledge of framework. The final aim of Scatter Search is to keep a set
of diverse and high-quality members. The background idea of the method tries to store beneficial information of global optima in both diverse and elite set of solutions by recombination of initial samples from the set, which can exploit this information. This strategy consider as an iterative process, where a population of diverse and high-quality members that are divided into subsets and linearly recombined to generate weighted centroids of sample-based neighborhoods. The recombination results are find tuned by an internal heuristic and it will be evaluated whether or not they are taken.

Tabu search (TS) is a local search approach, which starts from an initial point and moves forward by altering the structure of variables, one at a time. Before acquiring the best solution as a starting point for the next step, the total or a portion of the neighborhood is assessed. The possible solutions, which have already been evaluated, keep in a temporary archive. This approach avoids re-selecting of Tabu solutions [26].

One essential complexity in structural optimization is dealing with a mix of discrete and continuous variables. In section 2.3.1, a comprehensive description of old and new methods for mixed variable problems is introduced. This section is going to present metaheuristics as a powerful tool to handle mix variable problems. Metaheuristic techniques are global optimization methods that attempt to reproduce natural phenomena or social behavior, for example biological evolution, stellar evolutions, thermal annealing, animal behavior, music improvisation [38, 139]. Two main characteristics of metaheuristics are distinct from other optimization algorithms: exploration and exploitation [99, 140]. Exploration guarantees a full search of the whole design space and avoids being stuck in local optima. It is also called the diversity character of algorithm. Higher amounts of exploration properties lead to slow convergence of optimization algorithms. Next property, exploitation is the ability of the algorithm in local search around the current optimum member. A suitable balance between exploitation and exploration increases the performance of algorithms in terms of reliability, accuracy, and fast convergence. Different methods of balancing are introduced in literature and diversity control is one of the most favorable ones. In chapter 3, proposed methods to maintain the diversity of optimization algorithms are introduced. Biologically inspired algorithms are one of the main categories of the nature-inspired metaheuristic algorithms. More specifically, these algorithms are based on the selection of the fittest in biological systems, which have evolved by natural selection over millions of years [38, 39].
2.2.3 Hybrid metaheuristics and meta-models

Depending on the number and type of variables, it is not always easy to make an exact decision between approximation and direct search methods. This chapter considers hybrid algorithms, which are made up by a combination of approximation and metaheuristics or metaheuristics together. Combinations of algorithms have been carried out for different reasons and in different ways. It could be difficult to select a suitable algorithm or find the optimum combination of algorithms when different optimization problems have to be facing [101, 108]. In this state, some adaptive selection models are proposed to intelligently change the probability of existence of algorithms or the activation of their internal operators [209, 211, and 238]. In chapter 3.2, an adaptive hybrid model will be introduced to optimize the geometry and material combination of a multi-material structure which works with the best advantages of available algorithms. Talbi [39] proposed a taxonomy based on the two design issues: functionality and algorithmic architecture.

![Classification of hybrid metaheuristics, according to Talbi [39]](image)

Raidle [52] categorized hybrid metaheuristics into four different groups. One of the best known categorizations is the order of execution. In batch models, an algorithm works separately after another one. The working direction of this type of hybridization is one way. It needs severe control of the exchanged information between algorithms to protect the genes information. However, the performance of the sequential hybrid model was good enough to be paid attention by many researchers. Coello [41] used a bi-level shape optimization to optimize a 3D-wing under two coupled load cases from aerodynamic and mechanical types. In the first step, coupled variables were reduced using Proper Orthogonal Decomposition (POD), which expresses any long vector as linear composition. In the second step, RSM and MLSM were employed to estimate the scalar coefficient of the POD model. Park [42] optimized both
geometry and material properties of some BIW parts with the aim of weight reduction while keeping the bending and torsional stiffness below the critical levels. He explained the reason for two level optimization as follows: a huge number of input variables $2^{195}$ leads to slow convergence. Using screening method reduces them to $2^{89}$ in the first step and the second step be continued with GA. He obtained the surrogate model of his problem, using of intensive DOE on simulation of different structures and used it for the second step. He compared the results of single level and two level optimization to show the performance of the proposed method. Chen [43] employed the bi-level method to optimize continuous variables like thickness of layers and discrete variables for stacking sequence optimization problems. To optimize discrete variables he used GA, and to estimate the model branch, multipoint approximation was employed. He mentioned the low convergence rate of GA because of long chromosome length. Haichao [44] in continuation of Chen’s [43] study, which optimized only stacking sequences, explained about the necessity of simultaneously size and laminate optimization. He used adaptive probability of mutation and crossover to optimize a composite cylinder, which was stiffened with vertical plates. To validate the ability of his bi-level method, he also optimized a satellite structure, which was made of honeycomb and T-shape stiffeners. Boyang [45] and Yamazaki [46] have accomplished similar studies by almost the same methodology.

Other widespread multi-level optimization algorithms are parallel algorithms. Cotta [53] introduced a comprehensive study on parallel algorithms and their classifications techniques. Firstly, he divided them into two main groups of single-solution and multiple-solution search algorithms. He then presented another categorization based on the nature and characterization of algorithms. Homogeneous and Heterogeneous are the most significant categories of this type of classification like, GA and Tabu search. Talbi [54] investigated three types of metaheuristics: i) MHs (Metaheuristics) with MHs ii) MHs with exact methods iii) MHs with machine learning and data mining. He recorded the performance of every algorithm in a Tabu list and then he dynamically changed the effect of more frequently used algorithms in the optimization process. Since this kind of hybridization introduces an algorithm combination that is more effective, this study is going to use almost a similar approach for hybridization. From Talbi’s point of view, three main groups could be determined in this field:

1- MHs to generate upper bound
In this method an estimation is considered to predict upper bound. Obviously, the time to solve for this type of problem will be less if the distance of Pareto optimal and Pareto front approximation is less.

2- Exact algorithms to explore very large neighborhoods
In this method exact solution are employed to find the large neighborhood from Pareto solutions. The main idea is to reduce the design space.

3- Exact algorithms to solve sub-problems
In this method, exact algorithms solve the sub-problems, which are generated by metaheuristics. The region that is recognized by MHs is searched in more detail by exact algorithms to find the global optimum which are probably located not so far away.

2.3 Challenges and complexities of structural optimizations

In chapter 2.1, types of structural optimization and related studies from literature was presented. In chapter 2.2, different kind of optimization algorithms were introduced and some more applicable ones were discussed in detail. This chapter, regardless of the type of optimization and its algorithm, classifies the challenges and complexities in size and material optimization problems. In every category, well-known approaches and solutions are described. The main goal is to develop a powerful methodology to overcome the difficulties which users face in the pre-processing step of structural optimization.

2.3.1 Mixed discrete/continuous variables

Gandomi [39] employed firefly algorithms (FA) to optimize mixed discrete [164] and continuous variable in some optimization problems. Based on some evolutionary algorithms, Dimopoulos [56] optimized some mathematical and mechanical problems consisting of both discrete and continuous variables. He compare his result with similar results of researchers in literature. During the categorization of different types of optimization algorithms, variables are divided into three groups, including discrete, continuous, and mixed variables [4, 12, 14, 104, and 215]. Haichao [44] used two-level optimization method to optimize continuous variables and discrete variables like the number of composite layers. The author employed the NSGA-II (non-dominated sort genetic algorithm) and epsilon constraint methods to optimize a generic BIW component consisting of continuous and mixed variables [242]. The number of composite lamina and the existence or non-existence of a part in assembly are considered as discrete variables in that study. Hvejsel [58] divided a huge number of different variables into convex and non-convex continuous relaxation which can be solved with gradient based methods. He used his proposed method to reduce the weight of plates under different load cases. Schumacher [253] employed an approximation method (LSM) to optimize a structure with nonlinear behaviors. He mentioned the number of initial samples to make the surrogate model play a significant role in the accuracy of the model.
2.3.2 Multi objectives problems

A multi-objective problem (MOP) is defined as:

\[
\text{Minimize/Maximize } \mathbf{F}(\mathbf{x}) = [F_1(x), F_2(x), ..., F_n(x)]^T, \\
\text{Subjected to: } \mathbf{g}(\mathbf{x}) = [g_1(x), g_2(x), ..., g_n(x)]^T \geq 0 \\
\text{h}(\mathbf{x}) = [h_1(x), h_2(x), ..., h_n(x)]^T = 0
\]

The vector \( \mathbf{x} \in \mathbb{R}^n \) is formed by a decision variable representing the quantities for which value are to be chosen in the optimization problem [55]. If there is no conflict between objectives or if increasing and decreasing a variable will simultaneously increase or decrease of all objectives, it will be possible to solve problem independently for every objective. But usually some objectives have different relations with the input variables. For instance reducing the thickness leads a lighter structure but no stronger one. Since the optimization of automotive structures that is the main subject of this study, dealing with many objectives and investigations of well-known approaches in this field will be useful.

1- Using preference relations that generate a finer solution order works better than Pareto optimality. Indeed, these relations provide further rank nominated results.

2- Reducing the objective numbers during the optimization process. Main objective of this method is to recognize the objectives which do not conflict and eliminated them. Brockhoff [59] in his study of reducing dimensionality, said that for evolutionary algorithms with less than four Pareto sets acceptable solutions are employed. But with an increasing number of objectives, the quality of Pareto become worst. In addition, decision-making methods could not be useful. He asked some questions which help to solve the problem of multi-objectives problems, e.g. whether all objectives are important to be optimized at the same time or in which situation reducing objectives is logic. How we can calculate the minimum required number of objective in many objectives optimization problems?

3- Scalarization method: the main idea comes from executing some single-objective along the search vector, which are identically distributed in objective space. In this approach, Pareto set estimation consists of optimums which are found by each single objective.

Coello [94], Marler [60], Garret [2], Ehrgot [3], Schumacher [1], Andersson [160], and Deb [5] have introduced a wide range of solutions to deal with multi-objective problems as follows.
**Weighted sum method**

This approach is defined mathematically as follows:

\[
\text{Min} \sum_{i=1}^{k} \omega_i f_i \bar{x}
\]

Where \( \omega_{i \geq 0} \) are the weighted coefficients representing the relative importance of objectives.

\[
\sum_{i=1}^{k} \omega_i = 1
\]

Changing of \( \omega \) for every objective gives different optimum results on the Pareto set. The weighted sum method considers as one the simplest methods of multi-objective problems. However, there is a weak point in case that the Pareto set objective values form a nonconvex behavior together [89]. Kim [64] proposed an adaptive weighted sum method to overcome above-mentioned disability. This approach concentrates on regions which have not been searched up to now using other weighted ratios.

**Figure 2.10.** Original weighted sum method

**Adaptive weighted sum method**

**Epsilon-constraint method**

Beside the weighted sum method, the epsilon-constraint method is considered as one of the best and well-known techniques in multi-criteria optimization. It reformulates the MOP by just keeping one of the objectives and restricting the rest of the objectives within user-specified
values [260, 313], see Figure 2.11. The epsilon-constraint method is mathematically defined as follows:

$$\min_{x \in \mathbb{N}} F_\mu(x),$$

Subjected to $F_m(x) \leq \varepsilon_m, \ m = 1,2, ..., m, \ m \neq \mu$

Where $\varepsilon \in \mathbb{R}^0$, the component $\varepsilon_\mu$ represents the upper boundary of the value of $F_m$ and does not necessarily mean a small value close to zero. One weak point of the epsilon-method is that the solution are quite dependent on the amount of epsilon vectors. Especially when many objectives are taken into account, the user need to prepare a lot of information to provide this vector.

![Figure 2.11. Epsilon-constraint method with two constraints](image)

Alvarado [71] employed the epsilon-constrain method to handle constraints in the optimization of grip dimensions using hybrid metaheuristics.

**Weighted metric method**

Another approach to convert a multi-objective problem into a single objective is the weighted metric method. In this method, summation of distance between a target objective and any other objectives result in the fitness value as mathematically defined:

$$\min_{x \in \mathbb{N}} F_\rho(x) = \left( \sum_{m=1}^{M} \omega_m |f_m(x) - Z_m^*|^\rho \right)^{1/\rho},$$

Subject to $g_j(x) \geq 0, \ j = 1,2, ..., j$;
The parameter \( \rho \) can vary between one and \( \infty \) and if \( \rho = 1 \), then the metric method is changed to the weighted sum method. When \( \rho \) is big enough problems convert to the minimization of the biggest \( f_m(x) - Z^*_m \). Figure 2.12 shows that Pareto set results are changed when \( \rho \) is changed.

![Weighted metric method, \( \rho = 1 \)](image)

![Weighted metric method, \( \rho = 2 \)](image)

**Figure 2.12.** Weighted metric method, \( \rho = 1 \)  
Weighted metric method, \( \rho = 2 \)

One benefit of this method is that it fits for nonconvex problems but one needs to define a proper value for the target objective. Otherwise, similar to the weighted sum method, there is the probability of artificially changing the effect of some objectives to others.

**Benson’s Method**

This method is similar to the metric weighted method with the difference of an ideal target construction. The ideal objective \( T^0 \) is taken randomly from feasible space. By this way, the fitness function calculated by summation of every distance is as follows:

\[
\text{maximize } \sum_{m=1}^{M} \max(0, (Z^*_m - f_m(x))) ,
\]

Subject to \( f_m(x) \leq Z^*_m \), \( m = 1, 2, \ldots, M \)

Figure 2.13 shows the Benson’s method. If \( Z^0 \) is correctly selected, then Benson’s method could be used for nonconvex problems, similar to the metric method.
The weakness of this method is the necessity of more constraints to limit the optimum space of results. As the obtained objective function is non-differential, it is not suitable for gradient-based algorithms.

**Value function method**

In this method, the user needs to define a mathematical model to express all objectives. Then, the remaining task is to find the maximum or minimum of this function. This method could be useful when enough information for the preparation of the function are available. The disadvantage of this method is the difficulty of function definition for problems, which an accurate function is not able to define.

**Goal programing method**

The main target of this method is to find some solutions that have already been achieved for one or more predefined objectives. If the objectives are not correctly defined, then the target will be the minimization of distance between every objective and predefined objective. To solve the goal programing, every goal is converted to at least one equality constraint and the target will minimize these deviations.

**Hybrid methods**

Using the advantages of weighted sum and epsilon-constrain method at the same time leads to better control of multi-objectives problems. For instance, objectives are converted to single objectives using the weighted sum method, and constraint reduces the design space during the iteration using the epsilon method.
Elastic constraint method

To eliminate the weakness of the epsilon method and scalarization approach, and to find efficient solutions, the elastic constraint method allows the constraints to penalize with an adaptive scheme. In this way, the fitness function is mathematically defined as:

\[ \bar{F}(x) = F(x) + r \sum_{j=1}^{m} \left\{ \max\{0, g_j(x)\} \right\}^q \]

The amount of p and r changes the violation of constraints in the fitness function. These parameters could be dynamically changed during the iterations [65].

Max-min formulation

In this method, the fitness function is individually constructed for every objective. Then, the solution is selected from minimum value of maximum distance as follows:

\[ P[F(x)] = \max\frac{f_j(x) - \bar{f}_j}{\bar{f}_j} \]

Where \( \bar{f}_j \) is the best target of every objective and \( \bar{f}_j > 0 \).

Beta-method

As a complementary method to the max-min approach, it is possible to minimize the beta in every iteration far objectives from the predefined value will be eliminated.

\[ P[F(x)]: = \beta \quad \text{with} \quad \left[ \frac{f_j(x) - \bar{f}_j}{\bar{f}_j} \right] - \beta \leq 0, \quad j = 1, 2, ..., m \]
2.3.3 Multi-disciplinary problems

This chapter explains another complexity in structural optimization problems, the so-called multi-disciplinary difficulty [140, 159]. Different approaches to deal with a couple of disciplines as the researchers' point of view are introduced here. Martinet [66] provided a comprehensive study and references of different architectures to solve multi-disciplines. He classified two main group of methods, including of coupled and uncoupled approaches. Balesdent [67] employed both coupled and uncoupled methods to optimize the mechanical and aerodynamical property of a structure. Coello [41], in order to optimize wind turbine, used a coupled approach to deal with different subjected disciplines. Considering a fatigue load case Michael [169] optimized a structure made of composite laminate. Nils [171] considered the nonlinear material behavior of composite laminate in his optimization problem. Oliver [229], under high level of load deviation, optimized a spring part made of composite laminate. Stoffel [8] considered both effects of elastic and thermal stresses in his shape/topology optimization as a coupled objective function. Schumacher [1] in addition to some common challenges of multi-disciplines in structural optimization, mentioned some other important issues in terms of company organization and infrastructure.

• Constructive procedures
• Geometry models used (CAD models)
• Simulation methods and models (different FEM, Multi-body simulation models …)
• Target functions and restrictions
• Available design variables
• Predictive qualities of the simulation models
• Types of sensitivity provision
• Calculation time for a simulation
• Parallel ability of the simulations
• Required hardware, for explicit FEM one needs high computing capacity, for implicit FEM it requires high storage volume
• Influence of scatters on the system
• Position of the discipline in the development process of the product
• Relevance of the discipline to the company
2.3.4 Handling of constraints

As shown in the previous chapters, finding appropriate strategies to deal with mixed variables, multi-objective problems, and multi-discipline problems have a great influence on the performance of structural optimization processes. In this chapter, another factor to increase the optimization quality is going to be introduced, i.e. constraint handling. Obviously, it is not imaginable to make a mechanical structure without any limitations and complexities. These restrictions include:

- Allowable limits to form sheet materials made of different properties
- Ultimate stresses that could be carried by parts and joints under different load cases
- Regulations and regional rules with regard to environmental or safety aspects
- Financial limitations of projects
- Time schedules of projects

Some limitations can be considered in structural designation before tooling design or the manufacturing phase. As an example, a manufacturer provides allowable bending angles of sheet metal parts for a specific thickness and material property as tables or graphs. Definitely, the designer will design cross-sections of the parts with angles equal or bigger than the preferred levels of the producer to avoid manufacturing difficulties or creating any defects in material. These kinds of restrictions are known as passive constraints and are easier to be implemented into the optimization process than the active constraints. Although, the acceptance level of active restrictions is determined before the design process, it is mandatory to find out their existing values in a FEM simulation or physical tests. As an example, the maximum allowable stress of a steel sheet has been announced as 390 MPa. The question is how many stresses are subjected to each surfaces of parts when it has been formed and assembled besides other parts in the structure? This kind of question could not be answered during the design process in CAD. It must be evaluated inside the whole structure during the validation phase. Here, it should be considered that some manufacturing limitations would only appear after the manufacturing process, however, they are already mentioned in the design process. As a well-known example, tearing and wrinkling of sheet metal parts in deep drawing processes can be located in active constraints group.

According to the group of constraints, there are different approaches of implementation in the structural optimization process. For dimensional constraints, based on the manufacturer’s data sheet, using of filtering method predicts the generation of infeasible geometries. Infeasible parts such as low strength parts could be deleted from the optimization process only after an evaluation step. Some manufacturing and production constraints in sheet metal parts have been considered in this study and are presented in section 3.2.
Giaccobi [68] employed a filtering method to assign appropriate materials from his material database to every part of a multi-material structure. To implement initial constraints, as manufacturing constraints, there are two well-known approaches including an explicit and implicit method, see Schatz [69]. Direct simulation and soft computing are employed for explicit and implicit approaches respectively. The direct simulation method with a fully parametrized model is coupled with the manufacturing process. This method shows a high accuracy in considering manufacturing limitations in structural optimization, but is a relatively complex and time-consuming procedure. For instance, it could be possible that by using Autoform [40], all regions with conditions close to tearing or wrinkling can be found and such parts can be eliminated from the optimization process. However, the implementation of manufacturing constraints with this approach seems applicable and quite exact, but needs a huge time-consuming FEM analysis because of running at least two kind of FEM software at the same time. Using a surrogate model instead of evaluation of all possible structures may lead this method towards applicability. Franke [155] combined casting process simulation with topology optimization tools to consider the manufacturing restrictions during optimization iterations. His methodology is considered as a direct simulation (explicate implementation) of manufacturing limitations in the optimization process.

In contrast, soft computing models which, based on collecting and modeling of experts and human knowledge, are relatively achievable compared to an explicit approach. Collecting the restriction aspect of every manufacturing process like stamping, RTM [134], SCM [135], injection, braided composite [49], and casting need massive efforts. After that, all information needs to be converted into qualitative and quantitative data to be useable in optimization algorithms. Schatz [69] prepared some questionnaires for production experts about the braiding process. They gave their opinions about the minimum allowable radius in bended surfaces and other aspect to improvement the quality of braiding process. Then he formulated and validated the obtained information based on the fuzzy logic method Figure 2.14.

![Figure 2.14](image-url)  
Figure 2.14. Architecture of a knowledge-based system including an interface to optimization program and manufacturing experts, according to Schatz method [69]
Until now, two groups of passive and active constraints in structural optimization processes were introduced. In the next step, circumstances of implementation in optimization processes will be discussed, regardless of the type of constraints. One explicit and fast approach to implement the constraints is to eliminate the structures consisting of the mentioned constraints from the optimization loop. This method recalls the Epsilon-constraint method as discussed in chapter 2.3.2 [260]. Nevertheless, when number and type of constraints are huge and the designer has no exact estimation of their values and dimensions, it will be necessary to carry out more accurate and intelligent approaches to assign constraints. In other words, inaccuracy in the selection of suitable algorithms to handle constraints leads to the rejection of a strong structure or accept an inefficient structure in the optimization process.

Coello [65] proposed a complete survey of handling constraints in optimization algorithms and classified them as follows:

- Use of penalty function
- Maintaining a feasible population by special representations on generating operators
- Separation of objectives and constraints
- Hybrid methods
- Novel approaches

The penalty function is the sum of all constraints, which originate from predefined limits, and which reduce the fitness of solution. Some well-known methods of constructing the penalty function are static, dynamic, annealing, adaptive, or self-adaptive approaches [107]. The model is divided into two parts consisting of satisfied and potential solutions. During the iterations, every solution remains an encounter, which is recorded in history and will be used in the fitness function of that member in next generation. Georgios [11] employed quadratic penalty function to consider manufacturing constraints in the optimization of a casting part using shape and topology optimization methods.
3. Proposed optimization algorithm in this study

3.1 Necessity of hybrid models in structural optimization

Introduction

This section introduces an adaptive hybrid algorithm for optimization tool, which will be employed to optimize a hybrid-material assembly of the automotive body in the coming chapter. The necessity of using hybrid algorithms becomes more important when the number of input variables is too big or objectives react with nonlinear response related to the input variables [85, 87, and 88]. During the last decades, parallel with growing of products’ requirements, wide ranges of new technologies have been developed in all industries. Both phenomena make the responsibility of designers too complex to develop new products. When the final price as one of the most important objectives is added to above-mentioned targets, the necessity for using a strong and fast optimization tools will be doubled. The question is how designers can be able to successfully find the best algorithm out of current optimization algorithms.

The optimum combination of variables that could be found by an optimization algorithm may be used to produce millions of parts for a couple of years. Finding a non-optimum design near to the global one, and use it in mass production alone may cause huge financial losses and drive to eliminate the producer from the market. A huge amount of metaheuristics and hybrid models have been developed during the last decades as well [39, 179]. Some of them are more than 50 years old and some of them are quite young and still under construction [83, 210]. Results of research on the performance of algorithms on test functions show that some of them are faster and some of them are more precise. The group of precise algorithms are usually slow in finding the global optimum. In contrast, the fast algorithms are not reliable enough and are normally converged to some local optimums. Evidence from literature shown, an optimum combination of algorithm features lead to develop better algorithms with better accuracy and reasonable speed without the weakness of individual ones [48, 52, 113, 117, 140, and 184].

Definitely, the amount of combinations, period of combinations and logic of combinations have a big influence on the performance of hybrid algorithms [51]. For this reasons, the main control of hybrid algorithms should intelligently adjust the algorithm parameters as an ECU in a hybrid automobile. In the best case, this adjustment should be implemented independent from the number and type of variables, objectives and constraints. Therefore, for the adjustment of algorithms, parameters will be adopted automatically without requiring the user's efforts. The
adaptive search approach, which is considered as a suitable strategy to find the best sub-algorithms, their internal parameters, and their communications are in the focus of this study.

In order to present the influence of algorithm combinations, some well-known academic challenges from literature will be introduced at the end of this chapter. Generally, it will be shown that using adaptive hybrid algorithms offers more reliability than the single algorithms at the same optimization time [165, 170]. It should be mentioned that the accuracy of structural optimization algorithms is the most significant specification of this class of optimization compared to the speed. To quote Yang [225]: “For multimodal problems, computational effort should focus on the global exploration search, rather than intensive local search”. Using surrogate models to optimize nonlinear problems needs relatively less CPU time as well [77, 81, and 82]. This time is part of the simulation of some randomly selected structures to make the surrogate model. Then, during the optimization loop amount of objectives and constraints will be extracted from the obtained model, not from FEM. In the following, the week and strong points of metaheuristics in structural optimization application will be presented and immediately related solutions to overcome these disabilities will be discussed. This investigation of weaknesses and types of solutions helps to correctly select the stronger algorithms and avoid dealing with weak algorithms.

Metaheuristics have been applicable because of their ability to handle complexities, noise, imprecision, uncertainty, and vagueness [177, 178]. Not every metaheuristic has all required specifications to be considered as an accurate and fast algorithm. Some of them depend on initial parameters, and wrong parameters lead to unreliability of such algorithms. For instance, a PSO algorithm can easily fall in a local optimum, which is located on the way towards a global optimum position [150]. Hybridization of algorithms is normally carried out to balance the exportation and exploration specifications [225]. Not considering the mentioned phenomena leads to a slow algorithm and permutation convergence respectively. Creating a suitable balance between exploitation and exploration may be implemented by using predefined parameters in the first of optimization algorithm. By this type of balance-controlling, the algorithm will not be able to cause unbalancing during the iterations. In contrast, an adaptive approach that is able to sense the current behavior of the algorithm and to change the related parameters could intelligently correct the direction of search scheme [208, 209, 211, and 212]. It creates a flexible algorithm for different types of optimization problems. An adaptive balance strategy between exploration and exploitation is one of the special specifications of the proposed hybrid algorithm in this study, which will be explained in next sections. Burke [115] introduced a comprehensive survey of heuristics and different levels of mixture, which helps engineers to find the optimum hybridization approaches by the help of two main approaches.
1- Approach based constructive low-level
2- Approach based perturbation low-level

Some popular combination methods of hybrid metaheuristics will be demonstrated here.

### 3.1.1 Hybrid metaheuristics

Some researchers in order to overcome the disabilities of algorithms, and which were mentioned on the previous pages, have used an operator improvement approach of metaheuristics. They believed that improvement or replacement of internal operators and their mechanisms enhance the local search speed without disturbing the global search abilities.

Tsai has used Taguch’s method between crossover and mutation operators in genetic algorithms besides the random exchange of genes [173]. He has tested his improved model with 15 benchmark examples and not only obtained optimal designs, but also better robustness compared to single algorithm results in literature. His method is considered as a cooperative method, which is comprehensively classified at the end of this section.

Yildiz has used the Artificial Immune Algorithm (AIA) as a global search scheme and some Hill-Climbing algorithms as local search schemes to increase his ability for hybrid algorithm search [175]. AIA takes the affinity of a member after the mutation step and replaces it if it is better than the original one. In the receptor editing stage, some members (genes’ combination or structures) will be randomly exchanged with initial values. Finally, the optimum result of AIA will be considered as the start point for the Hill-Climbing algorithm. This kind of hybridization is considered as sequential cooperation.

He has also combined a population-based algorithm named Differential Evolution (DE) with Taguchi’s method. In the first stage, he has used the ANOVA (Analysis of variance) approach to determine the sensitivity of the input parameters. By using of this information and Taguchi’s approach he constructed a population with the name of refined population. He used these strong members in the second step instead of randomly replacing of members in mutation and crossover. By this way, he increased the chance of reproduction of strong members and, as result, faster movement towards global optimum. He has used his proposed method to optimize a welded beam and compared the result with five other evolutionary algorithms. His method is considered as a sequential approach [167].

Yildiz has used a combination of bee colony optimization and Taguch’s method to overcome the limitations of big populations and the time-consuming problem and quality of global optimum results [174].
Adler has improved the genetic algorithm operators using of best members of a simulated annealing algorithm. By this way, he has enhanced two important properties of search algorithms. First, keeping the population number as low as possible, and second by improving the algorithm performance by adding a Hill-Climbing strategy [172]. Adler has obtained better performance by hybridization of an explorative algorithm (GA) with an explorative algorithm (SA) compared to a single search of GA [172]. In his study, he has used schedule annealing to control the probability of applying the mutation operator instead of accepting produced members.

Wang has introduced a combination of PSO (Particle Swarm Optimization [296]) and Bacterial Foraging Optimization (BFO) to optimize the crashworthiness of lightweight design. He has mentioned the weakness of PSO, which is permanently at risk of being stuck in a local optimum and has no specific approach to skip that risk. On the other hand, BFO, which uses randomized patterns to move the members is relatively slow in converging to a global optimum. Combining PSO and BFO allows to avoid the premature convergence of PSO and improves the search efficiency of BFO [165].

Rezaian [176] has improved the generating mechanism of new population by GA based on PSO algorithms. After generating a new population by PSO, which is carried out by movement of weak members towards strong members in PSO, he exchanged genes using crossover operator between candidate members. Then he applied a mutation operator on candidate members in order to use both operators of GA, namely crossover and mutation.

As a clear evidence from literature, a combination of different algorithms and their operators generate undoubtedly better and fitter members than the individual algorithms. However, the efficiency of combinatorial algorithms could be reduced when an inappropriate contribution of stronger operators decrease the influence of other operators.

Hybrid algorithms with sequential schemes are easier to implement than the parallel hybrid algorithms [48, 54]. However, there is always the risk of missing some good genes and combinations between algorithms. In contrast parallel hybrid algorithms offer, the chance to combine genes and operator generates it without missing the improvement by other operators. Alkhechafi has used another type of hybrid algorithm [131] by dividing the population into smaller sub-populations. She has used a new random walking positioning in Firefly to rebuild the new generated members by FA and Cuckoo Search (CS) [105] algorithms. “The superiority of this hybridization in parallel is to guarantee the search in the best location of previous iteration instead of having search or re-search in the random way”. She compared the performance of CS-FA in some test functions with the individual performance of FA and CS. She also used the movement operator of a PSO algorithm instead of mutation in GA to
intelligently search the design space [131]. She used her hybrid algorithm to optimize water quality evaluation. In other compositions, she used the movement operator of FA instead of mutation in GA. She obtained less time to optimize some benchmark functions compared to individuals’ ones by using this new composition [162].

Rahmani [161] did not only use the crossover operator for the recombination of the initial population, but also for improved members of FA. In the second iteration, two strong parents from the movement operator of FA are crossed over and will be sent to the next iteration with children. Shi [154] applied PSO and GA in parallel and independent by together. After satisfaction of stopping criteria, he sent the current population to GA and PSO again. In this type of cooperation, fitter members of both algorithms have the opportunity of finding stronger members to recombine.

By combining the advantages of both the firefly algorithm (FA) and differential evolution (DE), Zhang [153] has introduced a novel hybrid population-based global optimization algorithm, called hybrid firefly algorithm. Above-mentioned algorithms are executed in parallel to improve the communication of sub-algorithms and thus enhance searching efficiency. He has evaluated the performance of the proposed hybrid algorithm on two set of selected benchmark functions.

Kummar [152] introduced PSO as a provider for diversification and GA as an intensification in PMHs. In his hybrid algorithm, PSO find the local optimum and send it to GA for fine-tuning. The best-found member by GA will send it as a global optimum to PSO again.

A reciprocate usage of PSO and GA operator was applied by Wahed [151] to improve the performance of the initial population. He evaluated the members, determined the fitness of every member, and gave them a new position, using the PSO movement operator. These new positions are improved by GA in the next steps. During the iterations, he prevented infeasible members from entering next steps using a repairing system.

Kam [150] mentioned the premature convergence as a weakness of PSO algorithm. To overcome the mentioned disability he suggested three approaches as follows:

- Global best of PSO exchanges its genes with some fit members using crossover operation to tackle the monotony of best member.
- The stagnated member is mutated using the mutation operator.
- In a first iteration, GA and PSO work in parallel and improve the initial population. From the second iteration, PSO continuous alone and uses the best members of GA from the first step.

Ma [130] introduced a hybrid algorithm which uses the advantages of SA and GA. “Quick processing and exploration of large solutions space property of GAs and efficiency local solution improvement property of SA are taken into consideration. GA randomly generates the
solutions and SA further refines them”. A chromosome of GA is mutated and its fitness is compared to its initial value. Each one was better take into account as the start point of SA. Rest members improve in parallel by GA.

In Che’s study, GA in first iteration considers some better members as the starting point of SA in case the probability criteria were satisfied [129]. These members will go back again to the current process of optimization if they have better fitness functions.

To improve the random behavior of the mutation operator in GA, the local search ability of SA was used by Jao [128]. He also used an adaptive cooling technique inside the SA algorithm to control the convergence rate. Zeb [127] send a certain percentage of the best solutions in each generation of GA for further intensification and trajectory search to SA. Alfa [126] explained, “While the standard SA typically begins with random initial parameter setting, in his proposed solution, the best solution of GA is utilized as the initial configuration of SA”. That means GA send 50% of the best members after satisfying of stopping criteria to SA. SA explicitly searches around the best structures for the globally best structures.

Vietor [261] has employed a stochastic optimization method to obtain failure criteria of advanced materials, in which they differ essentially from those of conventional materials with conflicting objectives.

Urli has proposed a tuning procedure to update his hybrid algorithm parameters in combinatorial optimization [124]. Alhadad [123] mentioned that after a few iterations of GA, the population moves toward being stuck in a local optimum and diversity of population becomes less. In this time, the current population will transfer to SA, which allows uphill jumps to higher levels of richness and avoids from being trapped in a local minima. After a few iterations, the new population is returned to GA. Jonasson [122] used a sequential combination for GA and SA. After a few iterations of GA, the best optimum is considered as the starting point of SA. Based on Talbi’s [39] categorization, this type of hybridization is a high-level relay hybrid. Kefi [121] has combined Ant colony optimization and standard Particle swarm optimization and 2-Opt algorithm. The ACO algorithm can explore the search space, PSO optimizes the ACO parameters and the 2-Opt reduces the probability of falling into a local minimum. Yoges [120] sent the best and worst member of GA to SA and then return back again to GA. His method is considered as a sequential approach. According to Chin [119], the hybrid algorithm advances some generations with GA, then the whole population is sent to SA for further improvement. The improved population will be considered as the initial population of GA later. Hart [118] suggested some local search algorithms beside GA to improve the output members of GA. He categorized two groups of local search algorithms consisting of random and adaptive random algorithms and he put SA in second group. In 2002 Talbi [179] introduced the most comprehensive categorization of hybrid metaheuristics. After that in 2006, Raidl [52]
introduced more detailed categories than Talbi and in addition developed some new criteria compared to Talbi’s classification.

Both of them, Talbi and Raidl, believed that the hybridization could be implemented in two levels. The first level is the high-level or weak coupling, in which algorithms more or less keep their initial specification. Of course, the genes’ information are exchanged between algorithms. As an example, the crossover operator maintains the exchange of genes between two parents and does not replace it with other operators or improvement methods. This type of hybridization could take place parallel or sequential. Both are called collaborative HMs.

The second level of hybridization named low-level or strong coupling, in which operators have not their original properties [106]. For instance, crossover may change with another operator and form a different algorithm. So, it is hard to say which algorithm is being implemented.

Jourdan [110] in 2009 extended Talbi’s research and presented another survey of MHs and exact methods. In 2012, Masrom [111] mentioned that the implementation of low-level hybrid algorithms is more complex than of the high-level ones and he introduced a comprehensive taxonomy of low-level hybridization. Alba [102] and Crainic [96] presented applications, recent advances and new trends of parallel metaheuristics.

Rodriges [114] in 2012 introduced exclusively a taxonomy for evolutionary and simulated annealing algorithms. Firstly, he divided the literature into collaborative (high-level) and integrative (low-level). Inside every group, he called those algorithms that worked as a group teamwork, and those that worked one after another relay. In the relay method, the output information of one algorithm is considered as the input information of another algorithm. Rodriges has investigated more than 400 metaheuristics from 1992 until 2011 about hybrid and resulted that more than 70% of HMHS have used SA as hill climbing algorithm, see Figure 3.1 (other hill-climbing algorithms are listed as greedy randomized adaptive search procedure, iterated local search, variable neighborhood descent, iterated greedy, and Tabu search).

![Figure 3.1](image.png)

**Figure 3.1.** Research considering SA as hill-climbing and other algorithms [114].
3.1.2 Adaptive hybrid metaheuristics

Up to now, the survey of recent studies on hybrid metaheuristics have shown that it is a powerful tool in optimization problems through recent literature. Hybrid algorithms also enable that collaboration or integration between algorithms or operators are not static, but dynamically updated during the iterations.

This automatic control offers the possibility of optimal usage for more effective algorithms and improve the capability of weaker algorithms. This specification of hybridization enables to obtain more robustness and quickness of hybrid algorithms.

Hasancebi [208] designed an adaptive dimensional search with the idea of changing the algorithm parameters during the search to achieve a rapid convergence and reliable results. He suggested three stagnation control mechanisms to escape local minimums.

- First an uphill move strategy when a stagnation is sensed. Then algorithm changes the rule of elitism members and allows non-improve members to participate in the new generation.
- Second an annealing strategy when stagnation is detected. A quasi-annealing approach is used to reduce the probability of acceptance for elitism members. During the iterations, Hasancebi uses an exponential formula to calculate the acceptance probability of a new member with better fitness.
- Third, penalty relaxation strategy. The main idea comes from reducing the violation of rejecting the infeasible members to search more design spaces. This job is done by altering the violation ratio from one to 0.95. However, the members who do not satisfy the problem constraints, may be let to come inside but the risk of local minimum is reduced.

Jankee [209] proposed a distributed adaptive MH selection that selects the most tailored MHs during the optimization search from a portfolio of available metaheuristics. He examined different approaches of choosing the MHs. Some MHs exchange their members in a migration phase at the same time like islands connection.

Candan [213] updates the migration rate during the iterations, using reinforcement learning principals. He used a semi-mutation operator to dedicate more opportunities to more hopeful algorithms.

Taha [211] suggested a combination of harmony search algorithm as a global search and some local search algorithms, which are automatically chosen during the iterations. SA, Great Deluge Algorithm, Record Travel Algorithm, Reactive Tabu search are his local search algorithms. He used two criteria, “impact evolution component” and “LS selection parameter” to choose the best LS algorithm as hill climbing algorithm. These criteria refer to the ability of LS algorithms to return better members to the base algorithm (harmony search algorithm).
Ya [212] designed a balance factor between PSO and Differential Evolution (DE) to control the contribution of every algorithm during the iterations. He took into account the lack of diversity as the most significant reasons for slow convergence rate. He examined the performance of PSO-DE with different values of balance parameters and compared the results.

**Conclusion**

Chapter 3.1 introduced the necessity of using hybrid metaheuristics with a comprehensive literature background in optimization problems. In addition, a wide range of combination approaches was demonstrated with related applications from chapter 2.2.3. In order to move towards the proposed methodology in this study, a brief summary of hybrid metaheuristics and their classification is presented in, Table 3.1.

Accurate, quick, and reliable optimization of mechanical structures need intelligent and flexible sets of rules to supervise the optimization algorithms. Between the wide arrays of algorithms, hybrid metaheuristics have obtained the first position as referenced in this field. Hybrid algorithms do not only experience a new combination type every day, but also receive new optimization algorithms [83]. However, the variety of hybrid algorithms offers a most amount of solutions to engineers, and finding the best mixture of hybridization appears as a difficult task. Therefore, two types of popular metaheuristic algorithms, SMHs (single-search metaheuristics) and PMHs (population-based metaheuristics), from well-known studies have been chosen to participate in hybrid algorithms of this study. Then, attendance or absence of operators or algorithms and their communications will be controlled, using automatic adaptation parameters. This specification, which is rarely seen in hybrid optimization algorithms, enables high performance and flexible forms of hybridization.
Table 3.1. Classification of hybrid metaheuristics as collaborative and integrative approaches. Related references and applications have been introduced previously.

<table>
<thead>
<tr>
<th>Level of hybridization or control strategy</th>
<th>Title of hybrid metaheuristic algorithm</th>
<th>Reference</th>
<th>Case study</th>
<th>Software</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collaborative</td>
<td>Modified Artificial Bee Colony + LS</td>
<td>[71] Alvarado</td>
<td>Control of a robotic hand (gripper)</td>
<td>Matlab 2013a</td>
</tr>
<tr>
<td></td>
<td>Modified differential evolution and Hook of Jeeves (MDE+HI)</td>
<td>[108] Huizhi</td>
<td>Mathemathic test cases</td>
<td>Matlab 2013a</td>
</tr>
<tr>
<td></td>
<td>Improved harmony search and Hook of Jeeves (MDE+HIS+HI)</td>
<td>[108] Huizhi</td>
<td>Test functions</td>
<td>Matlab 2013a</td>
</tr>
<tr>
<td></td>
<td>PSO, modified DE and Hook of Jeeves (PSO+MDE+HI)</td>
<td>[108] Huizhi</td>
<td>Test functions</td>
<td>Matlab 2013a</td>
</tr>
<tr>
<td></td>
<td>Multiple start guided neighbourhood search (MSGNS), GA+SA+Tabu</td>
<td>[113] Rama</td>
<td>Staking sequence</td>
<td>Matlab 2013a</td>
</tr>
<tr>
<td></td>
<td>The human immune system and artificial immune algorithm + Hill climbing</td>
<td>[175] Yildiz</td>
<td>3-points bending, machine tool spindle design</td>
<td>Matlab 2013a</td>
</tr>
<tr>
<td></td>
<td>Differential Evolution + Taguchi’s method</td>
<td>[167] Yildiz</td>
<td>Welded beam problem</td>
<td>Matlab 2013a</td>
</tr>
<tr>
<td>GA + SA</td>
<td>[172] Adler</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>PSO + Bacterial Foraging Optimization (BFO)</td>
<td>[165] Wang</td>
<td>Crashworthiness</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>PSO + GA</td>
<td>[154] Shi</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + PO</td>
<td>[151] Wahed</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + PSO (Type 3)</td>
<td>[150] Kann</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA</td>
<td>[130] Maheswari</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA</td>
<td>[129] Chen</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA</td>
<td>[127] Zeb</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA (cyclic GA-SA)</td>
<td>[126] Affarisy</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA</td>
<td>[124] Sanjay</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
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<td>GA + SA</td>
<td>[125] Alhedad</td>
<td>TSP</td>
<td>Matlab 6</td>
<td></td>
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<td>GA + SA</td>
<td>[122] Janasson</td>
<td>University Course Timetabling Problem</td>
<td>Matlab 6</td>
<td></td>
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<tr>
<td>GA + SA</td>
<td>[121] Panpan</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA (best and worst members of GA sent to SA)</td>
<td>[120] Yogeswaran</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA (several iterations of SA then SA)</td>
<td>[119] Chiu</td>
<td>Reservoir system</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + SA (random and adaptive random)</td>
<td>[118] Eugene Hart</td>
<td>Test functions</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>GA + artificial immune system (AIS)</td>
<td>[218] Bernardino</td>
<td>Academic Optimization problem</td>
<td>Matlab 6</td>
<td></td>
</tr>
<tr>
<td>Integrative</td>
<td>Hybrid Taguchi-genetic algorithm (HTGA)</td>
<td>[173] Tsai</td>
<td>Test functions</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>PSO + GA</td>
<td>Rezaiean</td>
<td>Test functions</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>GA + FA</td>
<td>Elkhechafi</td>
<td>Test functions</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>Cuckoo search + Firefly</td>
<td>Elkhechafi</td>
<td>Test functions</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>GA + PSO (using of PSO instead of mutation)</td>
<td>Elkhechafi</td>
<td>Water’s quality evaluation</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>FA + GA</td>
<td>Rahman</td>
<td>Test functions</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>PSO + GA</td>
<td>Slosodia</td>
<td>Transmission network planning</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>GA + PSO (Type 1, Type 2)</td>
<td>Kann</td>
<td>Test functions</td>
<td>Matlab R2009</td>
</tr>
<tr>
<td></td>
<td>GA + SA (adapted cooling)</td>
<td>Jeong</td>
<td>Test functions</td>
<td>Matlab R2009</td>
</tr>
</tbody>
</table>
3.2 Adaptive hybrid metaheuristics algorithm in this study

Introduction

Strong and weak points of exact search and evolutionary algorithms to find the local and global
optimums were introduced and discussed in the previous chapter.
In addition, it was illustrated how it is possible to combine two or more population-based
metaheuristics (PMHs) and single-search metaheuristics (SMHs) algorithms in order to cover
their weak points. It was shown that the hybrid algorithms have better performance to find the
global optimum without the risk of dropping in local optimum solutions.
During recent years a wide range of PMHs have been used in structural optimization problems
like global search, e.g. genetic algorithm, particle swarm optimization, differential evolution,
ant colony optimization, firefly algorithm, bee colony algorithm, and cuckoo search. Besides
PMHs, a wide range of single-search algorithms (SMHs) as simulated annealing, Tabu search,
and greedy search have been used for local search propositions [125, 133, and 180].
In this study, genetic algorithm (GA), firefly algorithm (FA), and differential evolution (DE) as
well as simulated annealing (SA) algorithm are used to enhance the capabilities of structural
optimization algorithm.
This chapter introduces a supervisor algorithm, which adaptively control the activity of some
algorithms and their relations by measuring the current performance of the whole algorithm.
This adaptive hybrid metaheuristic algorithm is named “AHMA” in the next sections.
Before introducing the adaptive hybrid metaheuristic algorithm of this study, a brief explanation
of GA, FA, DE, and SA will be presented. This brief review helps to understand the initial ideas
and designation of hybrid algorithm tool of this study easier.
Genetic algorithm

Three main operators handle the genetic algorithm (GA) to improve the initial population towards a global optimum: crossover, mutation, and elitism [228, 231, and 95]. Figure 3.2 shows a flowchart of standard GA and a schematic depiction of GAs’ operators for an ordinary stacking sequence optimization, published by the author [239].

**Figure 3.2.** Standard flowchart of genetic algorithm and two types of crossover and mutation
Firefly algorithm

Two mains parameters play the most significant role in firefly algorithm to enhance the quality of the initial population towards a global optimum: variation of light intensity and attractiveness [4, 38, 93, and 132]. Figure 3.3 shows a flowchart of FA and its related formula.

![Flowchart of Firefly Algorithm](image)

* is determined by its brightness that is related with the objective function

**Cartesian distance between member (i) and (j):**  \( r_{ij} = ||x_i - x_j|| \)

New position of genes for a weaker member (i) towards stronger (j):  \( x_i^{t+1} = x_i^t + \frac{\beta_0}{1+\gamma r^2} (x_j^t - x_i^t) + \alpha \varepsilon_i \)

Where:

- \( x_i^{t+1} \): new position of moving member
- \( x_i^t \): last position of moving member
- \( \beta_0 \): initial attractiveness of two members
- \( \gamma \): random number
- \( r \): distance of two members
- \( \alpha \): random number
- \( \varepsilon_i \): random vector from Gaussian distribution

**Figure 3.3.** Standard flowchart of firefly and calculation of new position for members
Differential evolution algorithm

Three main operators create the new population from the initial population in DE algorithm; crossover, mutation, and selection [47, 91]. Figure 3.4 shows the flowchart of a DE algorithm and its related formulations.

* $F$ is the scaling factor, which controls the magnitude of differential variation

** Noisy vector $= x_4 + F \times (x_3 - x_2)$

New member $= \begin{cases} 
\text{Trail vector} & \text{if } \text{fitness}_{\text{trail}} \leq \text{fitness}_{\text{member1}} \\
\text{member 1} & \text{otherwise}
\end{cases}$

Where:

1, 2, 3, and 4 are the members which are randomly selected in every iteration until new population is filled as the number of initial population

Figure 3.4. Standard flowchart of differential evolution and calculation of new members
Simulated annealing algorithm

Two main parameters have the biggest influences in SA algorithm to find a better solution than the initial one: acceptance mechanism and cooling scheme [80, 90]. Figure 3.5 shows the flowchart of SA and its related formula.

![Simulated Annealing Flowchart](image)

- **Boltzmann constant**
- **$D_t$** is decrement step of $T$ (temperature)

\[
T_{k+1} = T_k - \frac{T_{\text{max}} - T_{\text{min}}}{n - 1}
\]

In addition to the temperature condition other stopping criteria could be defined:
- Specific number of iterations
- Achieve specific improvement of initial member

*Figure 3.5.* Standard flowchart of simulated annealing and achievement of new member
3.2.1 Structure of adaptive hybrid algorithm in this study

A flowchart of the proposed algorithm in this study is presented in Figure 3.6.

Figure 3.6. Flowchart of adaptive hybrid metaheuristic algorithm in this study
Before explaining about the steps of the algorithm, it is necessary to introduce the control parameters’ vector. The control parameters’ vector consists of an array of numbers, and each number is responsible for adjusting and controlling an individual part of the algorithm. These parameters are located between predefined minimum and maximum values and they can be varied during the optimization loop. The amount of parameters depend on the predefined rules in the central part of the algorithm. Altering these parameters leads to increase or decrease of the number or existence of some algorithms or their communications during the optimization. Figure 3.7 and 3.8 introduce the control parameters of AHMA and their meanings.

![Diagram of control parameters]

**Figure 3.7.** Definition of control parameters, which automatically regulate the performance of the adaptive hybrid algorithm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Step</th>
<th>Initial Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ran</td>
<td>10%</td>
<td>80%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>SA, GA, FA, DE</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>mut</td>
<td>0.1</td>
<td>0.9</td>
<td>0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>α</td>
<td>0</td>
<td>1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>gam</td>
<td>5%</td>
<td>50%</td>
<td>5%</td>
<td>20%</td>
</tr>
<tr>
<td>fam</td>
<td>5%</td>
<td>50%</td>
<td>5%</td>
<td>20%</td>
</tr>
<tr>
<td>dem</td>
<td>5%</td>
<td>50%</td>
<td>5%</td>
<td>20%</td>
</tr>
</tbody>
</table>

**Figure 3.8.** Lower and upper level of control parameters

Note: second row (SA, GA, FA, DE) are discrete parameters, zero or one
The whole steps of optimization algorithm is implemented in a MATLAB scripts. Depends to
the complexity of optimization problem **Step 1** (generate initial population) and **Step 3 and 8**
(Evaluation) may be implemented in different ways. The remained steps of proposed
optimization algorithm in Figure. 3.6 are independent to the complexity of optimization problem.

For instance, to generate the initial population of sub-assemblies belongs to the car body a
parametric 3D model in a CAD software has to be constructed. However, to generate some
cross-sections for optimization of a cantilever beam there is no need for parametrization in
CAD. Similarly, to evaluate the mechanical properties of a sub-assembly belongs to car body
a CAE software has to be employed which is not necessary for a simple cantilever beam.

Here, the steps are explained in detail. Every step refers to the numbers which are given in the
block diagram in Figure 3.6.

**Step 1:** Algorithm produces equal number of initial population individually for GA, FA, and DE
using the MATLAB script.

**Step 2:** Initial values for control parameters are determined (Figure 3.8, column 5). User defines
these values in the MATLAB script at the start of optimization loop based on the followings rules.

“ran”: Initial value of “ran” which is defined the ranking percent of non-dominated members in
new population has been set on 10%. Because at the first iterations of optimization algorithm
there is no risk of uniform population. During the iterations, the amount of “ran” will be changed
based on the algorithm progress. Progress of algorithm is measured by four criteria (Step 13).
Circumstances of variation the “ran” value has been explain in detail in Fig. 3.15, 3.16, and 3.17

“SA”, “GA”, “FA”, and “DE”: They are “1” at the first iteration and that means they participate in
hybrid algorithm. When they introduce no efficiency during the iterations they become “0”, that
means they no longer participate in the next iterations with other sub-algorithms. Stopping
criteria for contribution of “SA”, “GA”, “FA”, and “DE” have been described in detail in Section
3.2.4.

“mut”, “α”, and “F”: which are the internal parameters of “GA”, “FA”, and “DE” have been initially
set on 0.9, 0.5, and 0.5. With these initial values, sub-algorithms introduce more exploration
properties at the first iterations. During the iterations they will be changed based on the
feedbacks of algorithm performance. Circumstances of changing the “mut” value has been
explained in Fig. 3.15. . Circumstances of changing the “α” value has been explained in Fig. 3.16.
Circumstances of changing the “F” value has been explained in Fig. 3.17.

“gam”, “fam”, and “dem”: defined the migration percent of members to other sub-algorithms.
They have been initially set on 20%. Because in the first iterations the sub-algorithms have no
need to improve with the help of others. During the iterations, every sub-algorithm, which have
accidentally no sufficient improvement, may need to be recovered with the help of lucky sub-
algorithms. Section 3.2.5.

**Step 3:** Initial population of every sub-algorithm is evaluated and the predefined objectives and
constraints are calculated and recorded. It needs to be noted, the amount of objectives and
constraints may be calculated using simple equations like weight and deflection of a cantilever
beam. However, in case of optimization of complex structures only using of FEM enables to find
the objectives and constraints like displacements, stresses, and energy absorptions.
Step 4: Predefined value of SA parameter in the first loop of the algorithm is adjusted to one. That means the simulated annealing algorithm will be executed. If during the next loops SA is changed to zero, the simulated annealing algorithm will not be executed and the currently evaluated populations are directly transferred to GA, FA, DE.

Step 5: The best member of GA, FA, and DE is selected and sent to SA in order to find the probably better neighbours. If the fitness of the enhanced member is better than the original one, it will be returned to the original algorithm. If more than three successive times SA could not be able to find a better solution or member, it will be interrupted by changing the SA value to zero. By this way, the whole CPU time of the hybrid algorithm will not be wasted because of an unfeasible algorithm.

Step 6: If the parameter values of GA, FA, and DE are one, their internal operators will recombine the evaluated members individually. They work actually in parallel and they send their best members to improve by SA in continue. Section 3.2.4 explains the contribution control of sub-algorithms in detail. The initial values of mentioned parameters are one and become zero if the mean value of its three last fitness value is less than others. There is another possibility of changing these parameters to zero; if no algorithm shows any improvement in its fitness seven successive times. Indeed, the unmotivated algorithm will be stopped to save the CPU time for others.

Step 7: If the value of “ran” is one, current population will be entered to ranking process.

Step 8: The recombined members (children) of algorithms from Step 6 will be evaluated and their related objectives and constraints recorded.

Step 9: As a percent of “ran”, some best parents and children (based on their fitness) move to the next generation. The initial value of “ran” has been adjusted to 10%. That means 10% of the next generation will be filled by best parents and children and crowded members of parent and children fill the rest. (Section 3.2.3).

Step 10: Three populations consisting of GA, FA, and DE members are sent to exchange their members through a migration process.

Step 11: A certain percent of “gam”, “fam”, and “dem” some best members of each algorithm will sent to another algorithm. Initially, these parameters are 5%. If the mean fitness of the sender algorithm is more than the receiver algorithm, then the migration percent of sender will be increased in the next iteration. It gives a chance to algorithms which are potentially strong, but have been unlucky to find good combination of genes. Thereby, if one algorithm has started with a poor initial population, it gets the opportunity to produce rich members once more.

Step 12: The stopping criteria will be checked and if satisfied, the whole hybrid algorithm will be stopped and the optimum combination introduced. Three stopping criteria have been considered here. 1- Achievement of a specific fitness value (must be defined at first) 2- receive to a predefined number of iterations 3- No improvement in fitness value after predefined number of iterations.

Step 13: Measuring of population’s diversity, individually for each algorithm. Checking the improvement of fitness value for the three last fitness values. Measuring the average of the three last fitness values of each algorithm.

Step 14: Updating of control parameters based on the measured outputs in step 13. For every output algorithm has one or more action plans to enhance the performance of sub-algorithms and hybrid algorithm as well. (Fig. 3.15 to 3.19 in Section 3.2.3, 3.2.4 and 3.2.5)
Until now, the flowchart and detailed description of the steps for the proposed algorithm in this study has been presented. However, some key points, like the Communication between sub-algorithms, manner of online measuring of the algorithm’s performance and circumstances of active actuators to keep the reliability of the algorithm, are still unclarified. In the following sections, these mean operators and techniques will be illustrated.

3.2.2 Sampling and choosing of initial population

One of the most important factors in the convergence rate and accuracy of population-based metaheuristics is the right choosing of the initial population [201, 206]. In general, the initial population should be as diverse as possible which the optimized design not being sensitive to initial population [39, 183, and 207]. To achieve the above objective, it is necessary to distribute the selected members uniformly over the whole design space without any trend to a certain regions. In other words, the members should be distributed as widely as possible to reduce the possibility of missing any design spaces.

Here, some well known and common approaches of initial population sampling are introduced. At the end, one of the most practical ones will be selected for sampling in this study.

**Random**: Based on mathematical theories, the members are randomly selected from the whole design space.

**Sobol**: Known as a deterministic method. In this method, the points are generated as far away as possible from their original structure [202].

**Cross validation**: Distributes the points in search space based on Kriging, which is used for response surface methods. In this approach, the error of model, should be received as close as possible to zero for the points, which have a better distribution on response surface [203].

**Latin Hypercube Sampling (LHS)**: This method is one class of sampling which is able to reduce the variance of Mount Carlo estimation method. The range of each input is divided into non-overlapping intervals of equal probability. One value from each interval is selected randomly with respect to probability density in the interval. Random pairing based on a pseudo-random number generated for all input variables are employed to formulate the final sample [204].

**Median Latin Hypercube Sampling**: Also divides each input variable into non-overlapping intervals of equal probability, but only the median point of each interval is selected.

**Hammersley**: It uses the Hammersley points to uniformly sample a(k-1) dimensional hypercube, and the results revealed that the Hammersley points provide the optimal location for sampling points so as to obtain better uniformity in the (k-1) dimension.
Pole investigated that well distributed sampling leads to increasing the robustness and avoiding of permutation. He has examined some sampling techniques on a non-dominated sort genetic algorithm II and has shown that weak sampling methods obtain wrong optimal results. He found out that the Kriging and Sobol methods for ZDT case problems with bigger dimensions achieve better results than the other approaches [201].

As an alternative to the diversity method, Pedro has introduced a center of mass criteria to select the initial population. He has explained that the appropriate sampling could be based on gene-level-diversity, chromosome-level-diversity, or population-level-diversity or a combination of them. From the computational point of view, executing all of them is infeasible and he has suggested population-level-diversity, which simultaneously measures the gene-level and chromosome-level [206]. In 52 test problem cases, Maarangen has investigated the effect of quasi-random-sequence sampling on the accuracy of optimum results [207].

Kiani, in order to find the 46 trailing points, and to make of his surrogate model for crash response of an automotive body, has used the Latin hypercube sampling (LHS) method [182]. Similar research has shown that the LHS method leads the acceptable and well-distributed samples of variable space in the construction of surrogate models [166, 168].

In the present study, the LHS approach has been used to choose the initial population for GA, FA, and DE algorithms. Additionally, LHS method will be used to generate the trailing points of the surrogate model for the side crash response model in section 4.4.4.

Figure 3.9.a, b: shows the distribution of 10 variables (X-axis), between 0 to 1 (Y-axis), in design space for five samples (five different colors), with two methods of sampling Figure 3.9-a random and Figure 3.9-b LHS methods.

**Figure 3.9.a.** LHS method has selected the variables from the whole space with a minimum-missed regions.

**Figure 3.9.b.** Random method has not selected the variables from the whole space and missed one big region in the top middle and in the bottom right area of the graph.
3.2.3 Assessing diversity and adaptive control of algorithm parameters

This chapter considers the significance of diversity in the performance of metaheuristics. Then, the measuring approaches of diversity will be presented. Thereafter, some methods to maintain the diversity in optimization algorithms is discussed. It will be seen that these approaches should firstly keep the diversity of population and secondly, in the case of lack of diversity, be able to moderate it. At the end of this section, an approach will be presented to measure, monitor, and control diversity of the optimization algorithm of this study.

A population is diverse when the combination of variables are uniformly distributed in the whole design space without any missed out regions. Figure 3.9.a considers a relatively diverse population and b has less diversification than a. Thereby, during the optimization loop, the weak members get the opportunity to get closer into the optimum design (strong exploitive property). In contrast, crowding of strong members takes the improvement possibility from weaker members, and the algorithm will probably be stuck in a local optimum (weak exploitive property). Therefore, an effective algorithm should make an appropriate balance between intensification and diversification properties to benefit both advantages. The tendency of choosing only the strong members called intensification leads to fast convergence probably towards a local optimum (Figure 3.10, green curve). Conversely, a low amount of intensification makes the convergence rate too slow and as result the algorithm become expensive (Figure 3.10, blue curve). Figure 3.10 shows the behavior of the genetic algorithm for two different amount of diversity and selective pressure (data from stepped beam problem, chapter 3.3.1).

![Figure 3.10](image)

**Figure 3.10.** Convergence behavior for a highly diverse population with slow convergence rate (blue curve) versus a low diversity population with fast convergence rate (green curve). Both of them are not reliable to find the global optimum.
Essentially, there are no special rules to establish a balance between diversification and intensification [225]. Those algorithms, which potentially have an adequate equivalency between exploration and exploitation, are considered as efficient algorithms. Diversity has been studied with two types of definition in literature. Genotypic diversity considers the unique members in terms of gene structure regardless to their fitness value. Phenotype diversity deals with members with same fitness values [222, 226]. Morisson has presented a unified method to measure the diversity that is suitable for measuring population diversity based on real parameters and binary-codes as well [221]. He has used the moment of inertia in his diversity approach, which provides a single method of computing for population diversity, and proved that it is more efficient than traditional methods for medium and big sizes of population. In this study, equation 3.1, which considers more effect of genotype than phenotype, is used to compute the diversity of populations.

\[
\text{Diversity} = 0.4 \times \text{fit}_{\text{div}} + 0.6 \times \text{str}_{\text{div}}
\]  

(3.1)

\[
\text{fit}_{\text{div}} = \text{number of members with unique deviation from min fitness/population size}
\]

\[
\text{str}_{\text{div}} = \text{number of members with unique distance from min position/population size}
\]

and min position is the minimum value of each variable or gene

distance of member \( x_i \) from min position \( x_j \), is defined as the cartesian distance: \( r_{ij} = \|x_i - x_j\| \)

Figure 3.11 shows the genotypic and phenotype diversity of a randomly generated population consisting of fifty members. The amount of \( \text{geno}_{\text{div}} \) and \( \text{pheno}_{\text{div}} \) obtained 58% and 56% respectively. Therefore, according to the equation 3.1, the diversity of the population will be determined as 57% (sampling and fitness are from the stepped beam problem, section 3.3.1).

**Figure 3.11.** Left diagram shows the deviation of 50 members in terms of genes combination and right diagram shows the fitness deviation of same population.
Until now, different types of diversity and an effective procedure to calculate them have been introduced [220, 222, and 224]. In the following, some well-known approaches to maintain the diversity will be discussed and, at the end, the proposed method for keeping diversity in this study will be presented.

Gupta [223] has offered a survey of maintaining the diversity in a genetic algorithm.

**Niching**: This method tries to achieve a natural emergence of niches and species in the search space. The niching method maintains the diversity and permits to focus on same local optima in parallel [227].

**Crowding**: In this method, members with same fitness values send a representative to the next generation. By doing so, the possibility of intensification in the new population will be reduced. Figure 3.12-b shows the qualified members who have been extracted from crowded members of Figure 3.12-a.

![Graph](image)

**Figure 3.12-a**: For an ordinary two-objective problem, distribution of first objective (f1) and second objective (f2) before implementing the crowding rule (20 members).

**Figure 3.12-b**: Distribution of first objective (f1) and second objective (f2) after implementing the crowding rule (only 12 members were selected, eight members have almost same fitness).

**Restricted mating**: Members who are located in predefined distance are selected. This distance is measured by the Hamming approach [185] and the permissible distance is determined before executing the algorithm. Therefore, members with almost the same gene structure have less chance to recombine together and produce a repetitive member.
Sharing: This method is a common and popular approach to maintain the diversity. Every member tries to share its fitness with its neighbors. The sharing method encourages search in an unexplored area of the design space [227, 228].

Ranked space: Two ranking methods are embedded in one approach, quality rank and diversity rank. The combination of these two methods supplies the best diversity to choose the new members [230].

Elitist: A number of members, without applying cross over and mutation processes are sent to the next generation. This procedure leads to maintaining the fitness of the whole population but may be induced to increase the strong members. Soremekum has suggested multiple selection to control the intensification of strong members [231].

Injection: Under a predefined rule and for specific iterations, some randomly generated members are sent to current population. Obviously, they should have a different structure and different fitness value from existing members [232].

Removal of genotype or phenotype duplicate: A simple and effective approach to eliminate the members with the same structure or fitness.

Fitness uniform selection scheme: In this method, one member is randomly selected between a minimum and a maximum fitness value. Then, one member with a fitness value close to this member is generated. On the way, some members have been produced between min and max fitness and show better diversity than in a standard method [233] (Figure 3.13).

Adaptive crossover and mutation: By changing the probability of crossover and mutation probability during the iterations, the amount of diversity can be controlled [234, 235, 236, 237, and 238].

All mentioned approaches try to keep the diversity of population that provides the possibility of searching unexplored space and escaping from local optima. As an example, Figure 3.13 shows the behavior of fitness uniform selection scheme (FUSS) methods [233].
Before presenting the circumstances of maintaining the diversity for the proposed hybrid algorithm, the procedure of ranking and crowding of a population will be introduced, as these two operators play a significant role in controlling the diversity, at least in this study.

**Figure 3.13.** Proposed methodology by Hutter [233] to escape from getting stuck in a local optimum. In the fitness uniform selection scheme (FUSS), all fitness levels remain occupied with “free” drift within and in-between fitness levels, from which new mutants are steadily created, occasionally leading to further evolution in a more promising direction.
This method, which was initially coined by Deb [186], provides a new population from a combination of population parents and children with two operators of ranking and crowding distance [87, 242], see Figure 3.14.

**Figure 3.14.** Generating the new population from a combination of parents and children (separating the dominated members and measuring the crowding distance for non-dominated members)

Rank-1 consists of members of the parents and children population that have dominated all members of Rank-2 in values of fitness. They fill the new population space as part 1 (Figure 3.14). Members dominated in the population Rank-2 are ranked based on crowding distance. Members who have more crowding distance than the others will go to the next step to fill the remaining space of the new population (Figure 3.14, part 2).

Crowding distance of $x_i$ member is calculated from min position $x_j$, as the Cartesian distance $r_{ij} = \|x_i - x_j\|$. Members with more crowding distance contain more different and more heterogeneous genes than the members with less crowding distance, and their presence in the next generation causes the expansion of search space or diversity. Thus, the algorithm is rescued from the risk of being stuck in local optima, which is one of the most common weaknesses of evolutionary algorithms.

It is important to note that in further iterations (after 10 iterations) despite choosing several survival plans, the deviation of gene structures becomes less and less. It will be difficult to fill the second box of the new population with too far crowding distance members. The algorithm
has an action plan to tackle this difficulty and fill the second part of the new population with no crowded members. The algorithm will inject some randomly generated members instead of near distance members.

Back again to the present algorithm in this study, the flowchart in Figure 3.15 shows the behavior of the algorithm in the case of sensing an undesired amount of diversity in the genetic algorithm. A detailed explanation of the process is introduced stepwise in following.

**Figure 3.15.** Detailed steps after step 13 (Figure 3.6) for the setting of control parameters in a genetic algorithm

Where:

- ran: ranking percent
- \( \text{step}_{\text{ran}} \): walk of ranking
- \( \text{iter}_{\text{cur}} \): current number of iteration
- mut: mutation probability
- \( \text{step}_{\text{mut}} \): walk of mutation
The flowchart in Figure 3.16 shows the response of the algorithm in the case of sensing a new amount of diversity in a firefly algorithm. The detailed explanation of the process is introduced stepwise after Figure 3.17.

Figure 3.16. Detailed steps after step 13 (Figure 3.6) for the setting of control parameters in a firefly algorithm

ran: ranking percent  
step_{ran}: walk of ranking  
iter_{cur}: current number of iteration

α: randomization parameter  
step_{α}: walk of randomization parameter
The flowchart in Figure 3.17 shows the response of the algorithm in the case of sensing a new amount of diversity in a differential evolution algorithm. The detailed explanation of the process is introduced stepwise after Figure 3.17.

**Figure 3.17.** Detailed steps after step 13 (Figure 3.6) for the setting of control parameters in differential evolution

- **ran**: ranking percent
- **step<sub>ran</sub>**: walk of ranking
- **iter<sub>cur</sub>**: current number of iteration
- **F**: scale factor
- **step<sub>F</sub>**: walk of scale factor
**Step 13:** From the main flowchart of the hybrid algorithm in Figure 3.6, in this step some performances of the algorithm are measured: Diversity of populations, improvement of the fitness value, and the average of the last three fitness values.

**Step-A:** The diversity of the genetic algorithm is measured based on equation 3.1.

**Step-B:** The improvement of the fitness value for the genetic algorithm is assessed by checking the amount of the last three fitness values.

**Step-C:** If the amount of diversity is less than 90%, that means the population is going to be filled with members with the same fitness values or the same gene structure. Therefore, the population is at risk to be stuck in local minima.

**Step-D:** If for three successive times, the fitness value shows no improvement, that means the algorithm could not be able to change its best members. The main reason is the lack of different genes in the structure of current members of the population.

**Step-E:** If any condition of Step-C or Step-D is satisfied, the algorithm has two plans to moderate it.

- First, decreasing of the ranking percentage by reducing the “ran” percent. As a result, fewer best members will be sent to the next population (“ran” is initially set on 10%). When the ranking percentage is reduced, the crowding members will be automatically increased and more weak members with a different gene structure will be present in next population (Figure 3.14).
- Second, for different algorithms:
  - In case of a genetic algorithm, the mutation probability is increased by increasing the “mut” value. As a result, more different genes will be imported to the population to provide the opportunity of improving the best members.
  - In case of a firefly algorithm, the randomization parameters are increased by enlarging "\( \alpha \)" and as result, members move around with greater steps and probably find different positions than before to scape of uniformity.
  - In the case of a differential evolution algorithm, the scale factor is increased by increasing “F” and as a result, more different gene structures are generated.

**Step-F:** If the amount of diversity increases to more than 90%, that means the population is going to be filled with members with different fitness values or different gene structures. Therefore, the population is at risk to converge too slowly because of exceeding the unfeasible members.

**Step-G:** If the condition of Step-F is satisfied, the algorithm has two plans to moderate it.

- First, increasing of the ranking percentage by inducing the “ran” percent. As a result, more best members will be sent to the next population (“ran” is initially set on 10%). When the ranking percentage is increased, the crowding members are automatically decreased and fewer weak members with different structures is presented in the next population (Figure 3.14).
- Second, for different algorithms:
  - In the case of a genetic algorithm, the mutation probability is decreased by decreasing the “mut” value. As a result, less different genes will be imported to the population to give more chances of participation to stronger members.
  - In case of a firefly algorithm, the randomization parameters are reduced by reducing "\( \alpha \)" and as a result, members move around with smaller steps and probably find better local optima than before.
  - In the case of a differential evolution algorithm, the scale factor becomes smaller by reducing “F” and as result; a less different gene structure is generated.
3.2.4 Contribution control of sub-algorithms in the hybrid model

This chapter firstly gives the reasons of using multi-algorithms in the proposed hybrid algorithm method. Then the detailed functionality of teamwork between these sub-algorithms in hybrid algorithms will be introduced. It will be shown how an efficient algorithm could help the inefficient ones to improve themselves.

Two groups of algorithms have been utilized in the hybrid algorithm of this study. The first group are population-based metaheuristics (PMHs) with the ability of searching the whole design space (explorative property) without the necessity of finding a good starting point. The second group consists of a well-known single-search metaheuristics (SMHs) with the ability of fast local search (exploitative property) with the necessity of finding a good starting point. SMHs helps PMHs to improve their intensification properties and PMHs help SMHs to improve their diversification properties by supplying good starting point.

In the first group (PMHs) consists of a genetic algorithm (GA), a firefly algorithm (FA), and a differential evolution (DE) algorithm. The genetic algorithm exchanges genes between members and could be able to find rich regions in the design space. In the classification of metaheuristics, GA is located in “memory based methods” algorithms [180]. FA is classified as “swarm intelligence method” algorithms which moves weak members towards stronger ones in order to find better combination of genes. DE is located in “memory based method” algorithms. The main strategy of DE algorithm is to generate an individual by calculating the vector difference between other individuals of the population.

In the second group (SMHs), there is only a SA algorithm, which behaves like an ordinary local search algorithm, but uses some randomization in the move selection to escape from a local optimum. Movements are accepted or rejected based on the probabilities taken from the analogy with the annealing process.

According to the literature evidences, PMHs have not enough motivation and agility to search around the strong members. Although, their strategy and operators have been designed in a way to improve the better members, this improvement is not fast enough. On the other hand, similar evidences show that if a single solution based algorithm like SA algorithm has a good starting point, it could be able to find a global optimum faster than PMHs. In this study, the SA algorithm is responsible for searching around the best-found members by GA, FA, and DE algorithms.
Contribution of Simulated Annealing (SA) algorithm

As mentioned in flowchart 3.6, step 4, the best members of GA, FA, and DE algorithms will be sent to the SA algorithm as the starting point. If the SA algorithm returns better members, it will be replaced instead of the weakest member of the original algorithm. If for three successive times, the SA algorithm is not able to return better members to the sender algorithms, it will be stopped by changing the parameter “SA” into zero (Figure 3.18).

![Flowchart of SA algorithm](image)

**Figure 3.18.** Stopping criteria for contributing of a sub-algorithm. Infeasible algorithms are stopped to provide more CPU power for efficient algorithms.

Contribution of GA, FA, and DE algorithms

In order to control the contribution of these algorithms in the hybrid algorithm, three control parameters have been defined: GA, FA, DE (Figure 3.8). In the first loop of the algorithm, these parameters have been set on one. That means GA, FA, and DE algorithms participate in the hybrid algorithm. There are two criteria to stop the contribution of PMHs in this study:

- No improvement of fitness for 7 successive times
- Average of the last three fitness values of one algorithm is less than the average of last three fitness values of other algorithms

If any of the above conditions is satisfied, related algorithms have to stop theirs activity. Figure 3.18 shows the process of sub-algorithms in the hybrid algorithm.
3.2.5 Migration of members between sub-algorithms

As a teamwork strategy, and in order to give more opportunity to inefficient algorithms, the sub-algorithms receive some best members from more efficient algorithms. The idea comes from the randomized behavior of PMHs algorithms. A population-based metaheuristic receives its initial population randomly, and most of its operators work randomly. Randomization may sometimes cause good combination of genes and sometimes not. A potentially strong PMHs algorithm may introduce itself as an inefficient algorithm, under unfavorable circumstances. Offering of some fit members may improve the efficiency of these algorithms.

In this study, the best members of the most efficient algorithm migrate to other algorithms. An efficient algorithm has the greatest average fitness compared to others. The migration percent of the most efficient sub-algorithms is increased through the iterations as \( step_m/iter_{cur} \). Where \( step_m \) is the migration walk (5%) and \( iter_{cur} \) is the current iteration number, Figure 3.19.

\[ gam = \begin{cases} 
\text{gam} & \text{if No} \\
\text{gam} + step_m/iter_{cur} & \text{if Yes} 
\end{cases} \]

**Figure 3.19** Migration of members between sub-algorithms, example from a genetic algorithm (GA) towards a differential evolution (DE) algorithm.

Where:

- \( \text{gam} \): migration percent of GA
- \( \text{step}_m \): walk of migration
- \( \text{iter}_{cur} \): current number of iterations
Conclusion

In this chapter, a flowchart of the proposed adaptive hybrid algorithm was introduced and described. It could be shown that the explorative properties of GA, FA, and DE algorithms besides the exploitative property of SA algorithms created a multipurpose hybrid algorithm with slightly global and local search abilities. In order to enhance the efficiency of the hybrid algorithm and make it an independent algorithm from the type and size of problem, a self-adaptive strategy was designed and implemented. These abilities maintain the diversity of populations, choose the optimal contribution of sub-algorithms, and coordinates the best communication between them. In the next chapter, the accuracy and reliability of the suggested hybrid algorithm will be assessed to solve an academical structural optimization problem.
3.3 Validation of proposed adaptive hybrid algorithm

In this chapter, with the aim of introducing more specifications of the proposed algorithm, a well-known academical problem from recently published literature have been selected to be solved. The optimization algorithm of this study has been implemented in MATLAB R2016a [156]. The optimization runs (not FEM simulations) have been carried out on a Laptop with Intel core i7-6700 HQ CPU 2.6 Hz and 8 GB Ram memory.

3.3.1 Stepped cantilever beam

The cantilever beam with five different sections, which is clamped from one side and is excited from other side, has been suggested by Vanderplaats [2], Figure 3.20.

![Cantilever beam with five different sections](image)

*Figure 3.20* Cantilever beam with five different sections [2]

Every segment has a width and height, which are considered as input variables, and consists of continuous and discrete variables as follow.

- \( b_1 = \{1, 2, 3, 4, 5\} \)
- \( b_2 = \{2.4, 2.6, 2.8, 3.1\} \)
- \( b_3 = \{2.4, 2.6, 2.8, 3.1\} \)
- \( b_4 \geq 1 \)
- \( b_5 \leq 5 \)
- \( h_1 = \{45, 50, 55, 60\} \)
- \( h_2 = \{45, 50, 55, 60\} \)
- \( h_3 = \{30, 31... 65\} \)
- \( h_4 \geq 30 \)
- \( h_5 \leq 65 \)

The single objective of the problem is to find the minimum volume of the beam as follows:
Minimize: \[ V = b_1h_1l_1 + b_2h_2l_2 + b_3h_3l_3 + b_4h_4l_4 + b_5h_5l_5 \]  

(3.2)

Following constraints have been considered:

The bending stress of each segment must be less than 14,000 N/cm².

\[ g_1 = \frac{6Pl_5}{b_5h_5^2} \leq 14,000 \frac{N}{cm^2} \]  

(3.3)

\[ g_2 = \frac{6P(l_5+l_4)}{b_4h_4^2} \leq 14,000 \frac{N}{cm^2} \]  

(3.4)

\[ g_3 = \frac{6P(l_5+l_4+l_3)}{b_3h_3^2} \leq 14,000 \frac{N}{cm^2} \]  

(3.5)

\[ g_4 = \frac{6P(l_5+l_4+l_3+l_2)}{b_2h_2^2} \leq 14,000 \frac{N}{cm^2} \]  

(3.6)

\[ g_5 = \frac{6P(l_5+l_4+l_3+l_2+l_1)}{b_2h_1^2} \leq 14,000 \frac{N}{cm^2} \]  

(3.7)

Maximum allowable deflection must be less than 2.7 cm.

\[ g_6 = \frac{Pl_3^3}{3E} \left( \frac{1}{l_5} + \frac{7}{l_4} + \frac{19}{l_3} + \frac{37}{l_2} + \frac{61}{l_1} \right) \leq 2.7cm \]  

(3.8)

Where: \( l_i = h_ih_i^3/12 \)

module of elasticity for all segments, \( E = 2 \times 10^7 \) N/cm²

Aspect ratio of height to width must be less than 20.

\[ g_7 = \frac{h_5}{b_5} \leq 20 \]  

(3.9)

\[ g_8 = \frac{h_4}{b_4} \leq 20 \]  

(3.10)

\[ g_9 = \frac{h_3}{b_3} \leq 20 \]  

(3.11)

\[ g_{10} = \frac{h_2}{b_2} \leq 20 \]  

(3.12)

\[ g_{11} = \frac{h_1}{b_1} \leq 20 \]  

(3.13)

Effects of constraints could be seen in the breakdown of fitness function, therefore this problem has been converted as an unconstrained optimization [2]. The fitness function (cost function), using elastic constraint method, can be written down as:
\[ F(X) = F(X) + r \sum_{i=1}^{m} \{ \max[0, g_i(X)] \}^q \]  \hspace{1cm} (3.14)

Where:

- \( F(X) \) : is the volume of beam
- \( r = 2 \) : constraint violation ratio
- \( q = 2 \) : constraint violation ratio
- \( g_i(X) \) : are constraints as mentioned in equation 3.3 until 3.13

The second term of equation 3.3 is the summation of all constraints. If one condition of equation 3.3 until 3.13 is satisfied, it will be violated by “r” and “q”. As a result, the unfeasible members will get a bigger amount of the fitness function and will have less chance to participate in the next generations. Obviously, reducing “r” and “q” leads to an acceptance of members which may have a smaller volume but have exceeded some constraints. Table 3.2 shows the minimum volume, and optimum input variables obtained by adaptive hybrid metaheuristic (AHMH) in this study. The comparison of evaluation numbers with similar studies shows that the AHMH of this study could find the global optimum with less number of evaluations.

**Table 3.2.** Proposed algorithm in this study found the minimum volume of beam less than the other methods and just 0.1% greater than study [112]. However, it found the global optimum with 50% less evaluations than the study [112].

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Method</th>
<th>Evaluations</th>
<th>b1</th>
<th>h1</th>
<th>b2</th>
<th>h2</th>
<th>b3</th>
<th>h3</th>
<th>b4</th>
<th>h4</th>
<th>b5</th>
<th>h5</th>
<th>Beam Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>[217]</td>
<td>Lemonge GA APW</td>
<td>35,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.289</td>
<td>45.626</td>
<td>1.793</td>
<td>34.593</td>
<td>64698.56</td>
</tr>
<tr>
<td>[218]</td>
<td>Bernardino AIS-GA</td>
<td>35,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.289</td>
<td>44.395</td>
<td>2.004</td>
<td>32.879</td>
<td>65559.6</td>
</tr>
<tr>
<td>[215]</td>
<td>Chen RNES f</td>
<td>12,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.311</td>
<td>43.108</td>
<td>1.822</td>
<td>34.307</td>
<td>64269.59</td>
</tr>
<tr>
<td></td>
<td>RNES 2</td>
<td>12,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.267</td>
<td>43.797</td>
<td>1.849</td>
<td>34.282</td>
<td>64322.43</td>
</tr>
<tr>
<td></td>
<td>RNES 3</td>
<td>12,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.348</td>
<td>42.804</td>
<td>1.783</td>
<td>34.753</td>
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</tr>
<tr>
<td></td>
<td>RNES 4</td>
<td>12,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.491</td>
<td>41.51</td>
<td>2.113</td>
<td>33.231</td>
<td>65416.9</td>
</tr>
<tr>
<td>[216]</td>
<td>Erbatur GA level 1</td>
<td>10,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.3</td>
<td>45.5</td>
<td>1.8</td>
<td>35</td>
<td>64558</td>
</tr>
<tr>
<td>GA Level 2</td>
<td>10,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.27</td>
<td>45.25</td>
<td>1.75</td>
<td>35</td>
<td>64447</td>
<td></td>
</tr>
<tr>
<td>[112]</td>
<td>Gandomi FA d</td>
<td>50,000</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.205</td>
<td>44.091</td>
<td>1.75</td>
<td>34.995</td>
<td>63893.52</td>
</tr>
<tr>
<td>This study</td>
<td>AHMH f</td>
<td>25,200</td>
<td>3</td>
<td>60</td>
<td>3.1</td>
<td>55</td>
<td>2.6</td>
<td>50</td>
<td>2.22</td>
<td>44.02</td>
<td>1.75</td>
<td>35.0</td>
<td>63947</td>
</tr>
</tbody>
</table>

- a Adaptive penalty method
- b Artificial Immune System (AIS) + GA
- c Rank-niche evolution strategy
- d Firefly Algorithm
- e Adaptive Hybrid Metaheuristic

Due to the random behavior of metaheuristics, it would not be completely fair to compare their efficiency with some successful runs. Here a comprehensive comparison criterion will be introduced, which consider not only the amount of global optima and evaluation numbers but also the reliability of the algorithm to find the global optimum.
The cost of analysis is the average number of analyses, which is needed to achieve a given number of reliability. Reliability is calculated by dividing the number of runs, which have found any given value of global optimum per total accomplished runs [187, 188]. For example, if 10 runs have been accomplished and 8 of them reached the given number, reliability will be 80% and the algorithm cost is the average number of applied analyses.

\[
\text{Cost} = \frac{\sum_{i=1}^{n} N_i}{n} P
\]

Where:
- \(N_i\): Number of applied iterations in the \(i^{th}\) run
- \(n\): Number of runs to obtain given "reliability"
- \(P\): Number of population

\[
\text{Reliability} = \frac{N_R}{n}
\]

Where:
- \(N_R\): Number of runs which found result less than FT

Where: FT: Fitness target value (using equation (3.14) with respect to the minimum values of weight, displacement and TSAIW criteria.)

The cost for similar methods (Table 3.2) for the optimization of stepped beam was not clearly available in all related literature. Thus, another study on costs of the proposed algorithm is shown. The optimization code has the ability to manually activate/deactivate sub-algorithms, ranking/crowding, migration, and adaptively control of whole algorithm, Figure 3.21.

**Figure 3.21.** Activation and deactivation of proposed hybrid algorithm functions for cost assessment (refers to flowchart of Figure 3.6). Furthermore, the effects of the improvements as presented in section 3.2.2 to 3.2.5 will be assessed by comparing their costs in stepped beam optimization (Table 3.3).
Table 3.3. Costs of different variations of proposed hybrid algorithm for stepped beam optimization

<table>
<thead>
<tr>
<th>Variation of Adaptive Hybrid Algorithm</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>OFF Parameters control</td>
<td>1.00</td>
</tr>
<tr>
<td>ON Simulated Annealing</td>
<td>0.93</td>
</tr>
<tr>
<td>ON Ranking / Crowding</td>
<td>0.90</td>
</tr>
<tr>
<td>ON Migration</td>
<td>0.95</td>
</tr>
<tr>
<td>ON Parameters control</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Note 1: when “parameters control” is off, that means:
- every sub-algorithm (GA/FA/DE) works separately
- without any cooperation with SA
- without using ranking technique
- without migration of members between sub-algorithms
- without controlling of diversity
- without adjusting the initial parameters of sub-algorithms
- average cost of GA/FA/DE has been considered

Note 2: When “parameters control” is on, that means:
- sub-algorithms (GA/FA/DE) work together and best members migrate between them
- sub-algorithms cooperate with SA
- ranking technique is carried out
- diversity is controlled and algorithm parameters will be updated

Note 3: To avoid conflict in the mutual effects of variations, only one variation has been activated at one time. Positive mutual effects of variation can be seen in the case of “parameters control” row.
Figure 3.22 shows the convergence rate of fitness for the hybrid algorithm; with and without parameters control (average of 100 runs are shown).

**Figure 3.22.** Adaptive hybrid algorithm adjusts algorithm parameters during the iteration with the aim of faster local search and escaping from local optimum. As a result, the algorithm will have a faster convergence rate.

Figure 3.23 shows the variation of the fitness value (Equation 3.14) through the iterations. Fitness value consist of volume of beam and constraints violation.

**Figure 3.23.** Variation of volume through the iterations for an ordinary optimization run.
Figure 3.24, 3.25, and 3.26 show the variation of variables (blue points) towards optimum values (green points) through the iterations for an ordinary optimization run. Green points show the optimum value of every variable, which was extracted from the best result of literature (Table 3.2). As variables have different amounts, the normalized value of every variable has been illustrated in the Y-axis of graphs. X-axis shows the width (b) and height (h) of the beam sections.
Figure 3.24. Optimal volume with respect to constraints after 5400 evaluations (eq. 3.3 to 3.13)
Figure 3.25. Optimal volume with respect to constraints after 19200 evaluations (eq. 3.3 to 3.13)
Figure 3.26. In iteration 42, the algorithm has been converged, because during the last seven iterations the algorithm showed no improvement in optimum result.
As explained in chapter 3.2.3, generating a suitable balance between intensification and diversification leads to enhancing both abilities of metaheuristics: local and global search. Some schemes have been introduced to maintain the diversity of populations during the optimization loops such as sampling technique, ranking, crowding, and adaptive control of internal sub-algorithms parameters. Figure 3.27 shows the deviation of diversity for an ordinary run of the stepped beam problem in two cases (diversity has been calculated from equation 3.1). The first case is a regular run without adaptive control of parameters and the second case with adaptive control of algorithm parameters (section 3.2.3).

**Figure 3.27.** The blue dashed curve shows that diversity reduces throughout the iterations for hybrid algorithm when there is no control of algorithm parameters (chapter 3.2.3). Therefore, the algorithm may be stuck in local optima. The green curve shows a nearly constant amount of diversity during the iterations, which has been provided by adaptive control of the algorithm parameters. It will reduce the risk of falling into a local optimum without reducing the convergence rate of the algorithm.
4. Implementation of the developed algorithm in structural optimization

Introduction

This chapter firstly explains the growing necessity of lightweight materials especially in the automotive industry. Then, it will introduce some reasons and evidences why a combination of traditional and lightweight material could be more suitable for design and for the cost point of view [189]. To find the best combination of old and new material in a multi-material structure, the requirements of structural optimization should be considered. Furthermore, some well-known difficulties and challenges of structural optimization methods are introduced. Four publications of the author related to the optimization of lightweight structures will be discussed. Finally, the chapter presents a proposal to overcome the complications of structural optimization. In order to show the suitability of the proposed method, it will be implemented into an automotive body to reduce the weight and increase the stiffness of one sub-assembly. Using material combinations besides variations of part geometries provides a wide range of opportunities for structural optimization. As optimization of structures would be meaningless without considering the manufacturing constraints, a comprehensive method for considering the manufacturing restrictions will be introduced.
4.1 Necessity of lightweight structures

By consuming about 30% of the overall energy in Europa, the transport industry is a significant producer of greenhouse gas emissions [190]. Automobiles have the largest share of those emissions. For this reason, the automotive industry strives to find solutions to reduce the effects of environmental destruction, which are created by its production processes. Increasing engine efficiency to reduce fuel consumption, designing more aerodynamic bodies, reducing the energy consumption of electrical components, developing hybrid cars and finally making the bodies lighter are considered the most significant methods to reduce the emissions. Recent observations show that reducing 100 kg of weight cause to reduce 8.5 gram CO₂/km [191]. Whoever, the current European directives state that the produced CO₂ in each 100 km must not exceed 130 gr. This acceptance level will be reduced to 95 gr until the year 2020 [192]. Weight reduction can be achieved by substituting conventional materials like steel with lighter materials like polymers and composites. Due to production limitations and also the final costs or even the maximum expected strength, a direct replacement with lighter materials is not easily applicable without considering certain conditions and restrictions. In most cases, it will be necessary to replace a certain percentage of original parts with new materials. In this way, a combination of two or more materials will be generated instead of one unique material. In this process, it is even possible that the geometry of original parts is changed. These challenges have asked for applicable strategies in recent years to make lighter structures by using different materials.
4.2 Motivation and challenges of structural optimization

One of the most efficient methods for weight reduction of sheet metal parts in the vehicle body is the replacement of conventional material like steel with new materials, which have a higher strength to weight ratio. Polymer-based composites are the best-known and most common types of light and strong materials in the automotive industry [193, 194]. There are two major concerns in the replacement process; one is finding the best combination of different materials and the other is the relatively high production cost of lightweight materials compared to conventional materials. Currently, it is six times more expensive, and optimistic forecast to be still three times more expensive by the year 2030 [193, 194].

For this purpose, it is necessary to place lightweight materials in the proper location of the vehicle without exceeding the set price limit, in addition to ensuring the strength. There are many parameters such as geometric dimensions and mechanical properties to take into account in order to meet the demands and expectations of a structure. On the other hand, the vehicle body is constantly under various and different loads such as vibration, crash, aerodynamic loads etc. Structural optimization techniques offer the possibility to provide the best combination of the input parameters in accordance with predetermined constraints and objectives for designers. Structural optimization methods are categorized in three groups: 1- topology of optimization, 2- form optimization, and 3- size optimization [195].

Taking into account all influential parameters and considering all loading conditions that may be imposed on the structure to find the best configuration of input parameters, increases the complexity and time of an ideal optimization algorithm. According to recent research, two methods or overall solutions have been provided for optimization of the mechanical structures: The first are gradient-based methods and approximation of objective function and the second are direct search methods such as evolutionary algorithms. This research provides a holistic method for the optimization of size and material combinations in multi-material structures under different loads on the vehicle body.

Independent from the above listed solutions, all structure optimization algorithms are involved with solving three categories of the following problems [2, 62]:

1- How to organize the variation of input variables
2- How to deal with multi-objective problems and handling of constraints
3- How to handle problems with a variety of loading and multi-discipline conditions

First category: Common variables in mechanical structures may be continuous or discrete. Variations of continuous variables such as profile dimensions, sheet thickness, and fiber
orientation of the composite laminate cause relatively predictable behaviors for structure properties such as strength or durability. Discrete variables [196], such as the number of layers of a composite part, material, and number of parts, are structure properties that any change of them causes relatively unpredictable sudden changes in the structure's behavior.

An exact model or a suitable approximation can be generated for continuous variables by sensitivity study, in order to explain output behavior to the entire space of initial variables. Schumacher [253] used the response surface for structural optimization with nonlinear behavior and least square method to fit with polynomial curve. The above describe method shows a good performance when variables are continuous and the number of problem objectives is not so high. However, finding behavioral models is difficult when there are discrete variables involved. It should be noted that multi-material structures normally engage a combination of discrete and continuous variables, where usually simultaneous optimization of geometry and stacking sequence are intended [197, 198].

Here, a brief overview on methods and strategies in the size optimization of mechanical structures is presented. Regardless of the type of optimization algorithm, finding the best dimensions of a sheet structure has been of interest to researchers in two groups [97, 199]. The first group is parameter-based and generates different geometries from initial design, for example by changing the length or angle of a line from the section profile [1, 200, and 243]. The use of CAD capabilities allows complex geometries are generated and evaluated by this method [244]. The second group generates different geometries for a profile by change of nodes located on a line or curve without having any parametric geometry [245]. These methods called, free-parameters, are involved in a large scale of the design variables and there is a need for a strong mechanism to prevent zigzag phenomenon in sections [199 and 198]. Due to mentioned weaknesses compared to the parameter-based method, the free-parameter method has not been in the focus of attention and investigation by researchers so much.

About the optimization of the stacking sequence as a problem with both types of continuous and discrete variables [312], it can be said that the problem has been studied with both gradient-based methods [246] and evolutionary algorithms [26, 214], such as genetic algorithm [247]. A description of the stacking sequence optimization results with response surface method and approximation method is given in references [248 and 249]. Meanwhile, in a research conducted by the author, a developed genetic algorithm for stacking sequence optimization in multi-material structures was used [239] and similarly introduced by [250, 231, and 254].

Fiber angle optimization can also be solved in two stages as a two-level approximation, which takes less time. Like Chen [43], who considered in the first phase a combination of variables
and in the second phase thickness of the layers, Jing [255] used the flexural stiffness parameters in direction of layers and optimization of maximum buckling load bearing.

The two-step method has also been investigated by researchers for problems that simultaneously to optimize geometry and fiber angle and has shown good results. Haichao [44] at first level used a genetic algorithm for estimating discrete parameters, and continuous parameters are optimized by mathematical estimation at the second level. Yamazaki [46], Boyang [45], and Allaire [76, 257] have provided similar research for simultaneous optimization of geometry and fiber orientation. Author [240] has conducted simultaneous optimization of geometry and fiber orientation and a combination of materials in multi-material structures of the vehicle body with genetic algorithm.

Second category: The second important issue that should be evaluated and analyzed in structural optimization with a powerful method is how to deal with multi-objective problems. Marler [60], Zhou [256], Ehrgott [3] and Coelho [41, 63] gave a complete description and comparison of this kind of problems and relevant solution methods. Pareto [258] initially provided the oldest method of dealing with multi-objective optimization problems. Then several other methods generally try to reduce the multi-objective problems to single-objective one and turn it into a constraint single-objective problem. This can be performed by the combination of objectives like the weighted sum method [60 and 89] or by assuming one of the objectives as the main objective and considering the rest as constraints like the epsilon-method [259 and 260].

The use of any of the above two methods has strengths and weaknesses that Ehrgott [3] had described. For example, the weighted sum method, while it is simple to run, requires appropriate and prudential definition of objective coefficients otherwise, the results may incline towards local optima. Also, the weighted sum method is not fundamentally able to identify solutions in nonconvex areas [28] Kim [64] provided an effective method to overcome this disadvantage.

Conversion of constraint optimization to non-constraint optimization by defining constraints as penalty function in the objective function is also widely used and effective methods to organize the effects of restrictions in optimization problem could be found. A complete description of the types of implementation of penalty function may be applied depending on the type of problem. For instance Coello described the penalty function with an adaptive ratio [65].

Third category: The third most discussed type in structural optimization concerns the demand for a strong and efficient strategy for problems, in which optimization may be performed not just for one type of load, but for a number and different types of load and boundary conditions. The complexity of the problem becomes severe when every single load (discipline) leads the
optimal values to a different way of the input variables space. For example, vertical walls in the section profile of a structure may increase the flexural strength of the structure, but will reduce fatigue durability.

Martinez [66] provides a comprehensive research from different architectures for solving multi-discipline problems. He divided multi-discipline solution methods into two categories: one discipline coupled with and the other disciplines independent of each other. Balesdent [67] investigated the design of space vehicles to satisfy the mechanical strength and aerodynamic efficiency with coupled and non-coupled methods and compared the obtained results. To optimize three-dimensional wings or blades, Coelho [41] also used the two-level reduction for coupled disciplines.

This study tries to introduce an optimal combination of methods in dealing with multi-variable, multi-objective and multi-discipline problems to optimize a multi-material structure. In choosing the right combination of existing methods, the following criteria play an important role:

1- Easiness of implementation on industrial level
2- Comprehensiveness and capability of use for similar problems without essential changes
3- Lack of restriction on the number and type of input variables, objectives and restriction of the architecture
4- Low cost or short run time of algorithm

The above criteria are subjectively assessed. Some of these criteria may be less important in other projects, while other criteria are more important. Therefore, they may need some modifications in the presented methodology for structural optimization in the subsequent chapters.
4.3 Proposed methodology for structural optimization in this study

In this chapter, an approach for the optimization of sheet made structures is introduced, which is able to find the optimum geometries of cross-sections and their materials regardless of the type and number of variables, load cases, and number of objectives. This method tries to present an appropriate solution for difficulties of structural optimization.

Actually, the methodology is based on generating a full parametric 3D model of the structure in CATIA [149] and evaluates the structures in ABAQUS [82] as two strong and well-known packages from 3DS Company. The automatic transfer of information between the software is coordinated by MATLAB [156]. To optimize the input variables, adaptive hybrid metaheuristics is used which was presented in chapter 3.2.

According to academicals and experimental studies by the author, the structural optimization approach could be divided into three main milestones.

- Realization of solutions
- Evaluation of solutions
- Optimization algorithms

Together, these three steps result in the optimization loop as shown in Figure 4.1.

![Figure 4.1](image)

**Figure 4.1.** Process of structural optimization in this study

Mostly, in every structural optimization problem, there is more than one concept or idea to consider. Therefore for multi-concept problems, an optimization tools presented here, which considers any number of concepts and tries to find a global optimum, Figure 4.2.
The author has used the illustrated method of Figure 4.1 and 4.2, in a couple of applications and published first results. On the next pages, four publications of the author will be presented to show different approaches in different applications of the structural optimization method.

**Figure 4.2.** Concept-based optimization proposed in this study
An approach for multi-objective optimization of hybrid material structure for mobility applications (ICCM20, Copenhagen, 19-24th July 2015)

In a first study, the stacking sequence of an A-pillar was optimized by using the developed genetic algorithm [239].

![Figure 4.3. A-pillar to be optimized using variations of cross-sections and materials](image)

A multi-material section for an A-pillar was suggested instead of a traditional full metal part. The inner parts of the A-pillar was planned to be replaced by a laminated composite part. A cohesive bonding method was used to connect the new part with the metal part, Figure 4.4.

![Figure 4.4. Construction of a hybrid A-pillar section](image)

A rollover accident was taken into account, which is assessed by a roof crush resistance test, according to the FMVSS No. 216a procedure, Figure 4.5.

![Figure 4.5. Test procedure of FMVSS No. 216a roof crush resistance](image)
The following variables were considered for the stacking sequence of the inner part:

- Minimum and maximum number of plies per laminate; 4 and 8 respectively.
- Available materials for plies are glass-epoxy and carbon-epoxy.
- Available thicknesses for individual layers are 0.5, 0.6 … 1.5 mm.
- No manufacturing limitation for ply orientation.

In order to generate an initial population for the genetic algorithm, the variation of the stacking sequences was coded as follows, Figure 4.6.

![Figure 4.6. Coding method of an individual layer in this study](image)

Minimization of weight and minimization of deflection while keeping the TSAIW criteria below one was considered as a multi-objective task. The assessment of the final result of two ranking methods showed that finding a local optimum instead of the global optimum is more probable in fitness-ranking than linear-ranking. For this reason, the linear-ranking method was used for pair selection as selection scheme. Two categories were considered to enhance the GA performance, first was the selection procedure and secondly developing of GA operators. In addition to the regular mutation, a similar operator was used to exchange the position of two plies of one child with a given probability. It was called LAMINA-SWAP which provided a chance to allocate the better position of layer with a different material and orientation. A comparative study on cost and richness of population was executed to compare the results of optimization with a regular form and the developed form of GA.

### Table 4.1. Performance comparison of different GAs (Reliability for all types is 90%)

<table>
<thead>
<tr>
<th>Type of GA</th>
<th>Cost</th>
<th>Richness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 GA-S*</td>
<td>412.6</td>
<td>0.484</td>
</tr>
<tr>
<td>2 GA-PR-PS**</td>
<td>372.4</td>
<td>0.658</td>
</tr>
<tr>
<td>3 GA-ME-PR-PS***</td>
<td>338.5</td>
<td>0.818</td>
</tr>
</tbody>
</table>

* GA-S: Genetic Algorithm with Standard operators
  Elitism probability = 0.03, Crossover probability = 0.7, Mutation probability = 0.1

** GA-PR-PS: Genetic Algorithm with PLY-RANK and PLY-SWAP operators
  PLY-RANK for all laminate group, Crossover probability = 0.7, PLY-SWAP Probability = 0.1

*** GA-ME-PR-PS: Genetic Algorithm with Multiple Elitism for selection method and PLY-RANK and PLY-SWAP operators
  N_e = 18 for ME selection and PLY-RANK for all laminate group, Crossover probability = 0.7, PLY-SWAP Probability = 0.1
Finding the best material combinations by multi-material joining, using genetic algorithm (ECCM17 - Munich, Germany, 26-30th June 2016)

The second study by the author in 2016 developed an approach to discover the best combinations of materials in hybrid material joining [240]. This method generated a wide possible choice of geometry and material combinations with any kind of number of parts in the structure. It also handled various geometrical and manufacturing constraints. The method consists of a fully controlled parametric modelling in CAD and optimization loop using a genetic algorithm, which coupled with CAE as follow:

2.1. Pre-processing step; generates and selects initial structures.

2.2. Evaluating step; calculates the fitness of individuals to satisfy the objectives.

2.3. Genetic algorithm; carries out the optimization loop. If the stopping criteria are not satisfied, it continuous, otherwise it will stop and display the result.

![Diagram](image)

**Figure 4.7.** Steps to generate the initial structures in CAD in optimization loop

An optimization of several objectives was carried out. Therefore the objective function was assumed as multi-objective function. A weighted sum approach was used to calculate the objective function.

Since evaluation of most engineering problems without considering existing limitations is meaningless, it was necessary to utilize an objective function to clarify the effect of constraints. Using an elastic constraints approach the constrained problem converted into an unconstrained problem, equation 4.2.

\[
\overline{F}(X) = F(X) + r \sum_{j=1}^{m} \left\{ \max \left\{ 0, g_j(X) \right\} \right\}^2
\]  

(4.2)
The adaptive penalty approach, which takes feedback from the search process, was used instead of the traditional static penalty method. That means that the second term of equation (4.2) could be written as follows:

\[
\lambda(t) \sum_{i=1}^{m} g_{i}^{2}(X)
\]  

(4.3)

Where \(\lambda(t)\) was updated every generation \(t\) in the following way:

\[
\lambda(t + 1) = \begin{cases} 
(1/\beta_1) \lambda(t) & \text{if case 1} \\
\beta_2 \lambda(t) & \text{if case 2} \\
\lambda(t) & \text{otherwise}
\end{cases}
\]  

(4.4)

Where case 1 and case 2 denote situations where the best individual in the last \(k\) generation was always case1 or was never case2 feasible, \(\beta_1, \beta_2 > 1, \text{and } \beta_1 \neq \beta_2\) (to avoid cycling). In the other words, the penalty component \(\lambda(t + 1)\) for generation \(t + 1\) decreased if all best individuals in the last \(k\) generations were feasible or increased if they were all infeasible. If there were, some feasible and infeasible individuals tied as best in the population, then the penalty did not change.

The B-pillar structure because of having a vital role in observing the side crash energy has been extremely improved and reinforced with various approaches. Figure 4.8.a shows the conventional appearance of a B-pillar and b, c, d show the possible alternatives with lighter and stronger geometries and materials. The overall length and position of every part was generated by fully controlled rules in pre-processing steps (parametric 3D model in CATIA).

![Figure 4.8.](image)

**Figure 4.8.** a conventional appearance, b, c, d some possible material variations
There are four different variations to generate different types of B-pillar structure:

1- Variation of cross sections: Regardless of how many parts the B-pillar structure has, it can generate different cross sectional profiles. These profiles must be located inside the fix and variable constrain which are defined at the beginning. The definition of the mentioned constraints was applied in CATIA and by dedicating random numbers inside the lower and upper levels the algorithm was able to generate different numbers of parts with various profiles.

2- Variation of material distribution: The individual parts of the B-pillar may have different lengths and different positions. This length and position are determined by two random numbers, so that every part can be shorter or longer and move in any direction.

3- Variation of Material: Wide ranges of material types are necessary to be stored in the material library which consist of complete specifications of every material. The material library must be divided into different categories based on the manufacturing technology. Any time the algorithm refers to the library to take a material from a given category, it generates a number between 1 and the number of materials in the mentioned group and chooses one material and assigns it in to a chosen part. The algorithm never refers to a category of material which is not related to the manufacturing technology of the selected part. Therefore, it can be ensured to assign the right material to every part of the structure.

4- Variation of parts’ numbers in structures: The algorithm had n number of multi-parts structure after concluding the previous step. In that study, n=100 and the possible number of parts was between two and four. According to randomly generated number 28 structures with two parts, 42 structure with 3 parts, and 30 structures with 4 parts created in initial population of GA.

In order to save the CPU time, four groups of structure were generated and evaluated in parallel. Also for the purpose of finding the optimum amount of variables, four parallel genetic algorithms were run simultaneously.

With the intention of taking into account the manufacturing constraints in optimization loop, the following strategy was used. Considering the wide range of manufacturing technologies and the number of needed operations to produce parts and assemblies was too complex to determine a similar and unique parameter to measure the manufacturing limitations. Because of that, complexity of production obtains by summation of all required costs of operations. For instance, tooling costs to make individual parts and assembly fixtures, machine investments, workers, and transportation represented manufacturing cost as a negative ratio for a four part structure.
Optimization of variable stiffness composites in the automated fiber placement process using evolutionary algorithms (CIRP CCMPM 2017, 8-9th June 2017, Karlsruhe)

The third study by author from 2017, is an approach to find the optimum position and the best length of layup dropping in AFP (automated fiber placement) technology [241]. As the objectives were minimum weight and maximum stiffness, the problem was considered as a multi-objective optimization. Fiber failure, matrix cracking, and onset of delamination were taken into account as the constraints for objective function. A comparative study was introduced to evaluate the performances of the genetic algorithm and firefly algorithm in finding the global optimum result.

Local patch reinforcement enabled to increase the mechanical abilities of structures using the AFP process. Figure 4.9 shows the schematic of an AFP process which drops tapes on predicted paths and with a specific feed rate and temperature to attach the patch to the substrate.

![Closed Loop Control](image)

**Figure 4.9.** Schematic of closed loop control during AFP process [251]

A comprehensive scheme was introduced to find the location and orientation of patches in problems, which are complex to solve and/or might have the risk of finding local optima with gradient-based or optimality approaches. Figure 4.10 shows circumstances of design space definition, objective generation, and handling the limitations. Optimum values for the requested variables are iteratively found through evolutionary algorithms (genetic algorithm, GA, and firefly algorithm, FA).
In order to increase the stiffness of an L-form part (Figure 4.10), a local stiffness reinforcement was aimed at without solely increasing the number of layers on the surface. A local stiffness improvement was achieved by attaching patches to tailored positions and orientations using AFP technology. Width and thickness of the patches were assumed to be constant and considered as 8 and 1 mm respectively. It was necessary to define dimensional parameters for the patches’ positions to provide the possibility of generating different lengths for the patches in every location. It was assumed that two individual patches would be enough to obtain the requested stiffness, else an additional patch would be attached on an existing patch position. A large surfaces and more complex load cases may require more patches per part, but did not make a big differences in the proposed method.

Figure 4.11 shows the necessary parameters to control the location of the first and second patch on a composite part.

**Figure 4.10.** Flowchart of proposed methodology in this study

**Figure 4.11.** Dimensional parameters to control the patches length and position
The minimum and maximum values of every dimensional parameter were determined so that patches did not exceed the part boundaries and secondly no patches were produced with a length less than a given value. By changing the input parameters between minimum and maximum limits, every sample took an identification which determines the location of the patch on the part surface. Figure 4.12 shows an ordinary configuration of patch location with related identification (Sample ID).

![Patch](image)

**Figure 4.12**. Parametrization of patches generates an identification code

A main objective of that study was enhancing the stiffness without considerably increasing the weight of the composite part. In order to simultaneously evaluate mentioned objectives, a weighted sum method was used. The part strength was assessed by determining the magnitude of maximum displacement at the force action point with ABAQUS. Stronger parts should a lower degree of displacement. Increasing the weight of the composite part by added patches was calculated by multiplication of patch length with the weight of unit of length. Fiber and matrix fracture and onset of delamination were considered as the restriction for every design. Any failure for any member dedicates a value to the third term of equation 4.5 and caused to increase the fitness value, and consequently reducing the chance of attendance in the next steps of the optimization process (normalized value of penalty was used).

\[
\text{Fitness} = \alpha U_n + (1 - \alpha) W_n + \text{Penalty}_n
\]  
\[\text{(4.5)}\]

$U_n$ and $W_n$ were normalized displacement and weight of part. $\alpha$ was the weight ratio of importance of objectives. Since variation of Layup parameters in AFP lead to a poor connection of patches to the ground parts [252], delamination defects were considered the main criterion to calculate the penalty. Figure 4.13 shows a delamination phenomenon which may happen due to 1) close distance of patch end to fix regions or force action point, 2) critical orientation of patch.
If shear stress ($\tau$) in any region between patch and ground part exceeded from the permissible shear stress value ($\tau_{\text{permissable}} = 50$ MPa), the algorithm added a penalty value proportion to the degree of violation as follows.

The following conditions were used as stopping criteria of the optimization algorithm:

- Maximum number of iterations
- Sensing no improvement of the best design
- Achieving to given target fitness

If the algorithm did not introduce a strong search ability during the iterations, it would be ended due to the second criterion. The amounts of the above mentioned stopping criteria were considered 30, 7, and 1.3. The amount of target fitness was calculated according to a maximum of 20% weight increase and at least 50% improvement of part stiffness.

The optimum position of patches is shown in Table 4.2 for both GA and FA algorithms.

**Table 4.2.** Optimum position of patches and performance of GA and FA

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Optimum configuration</th>
<th>Cost</th>
<th>Richness</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>$x_1$ 105 $y_1$ 28 $x_2$ 112 $y_2$ 62 $x_3$ 110 $y_3$ 22 $x_4$ 118</td>
<td>2645</td>
<td>0.89</td>
</tr>
<tr>
<td>FA</td>
<td>$x_1$ 32 $y_1$ 28 $x_2$ 111 $y_2$ 62 $x_3$ 110 $y_3$ 23 $x_4$ 117</td>
<td>2300</td>
<td>0.87</td>
</tr>
</tbody>
</table>

*Figure 4.13.* Loaded part (left), delamination onset (right, scale factor: 10)
The fourth study of the author dealt with minimization of weight and cost and maximization of strength and durability of a multi-material structure made of metal and composite as a multi-objective and multi-discipline problem. NSGA-II (non-dominated sort genetic algorithm) was implemented as a powerful algorithm in multi-objective problems for finding the best combination of discrete and continuous variables in structural optimization. Since no estimation about the minimum and maximum values of objectives was present, the use of the weighted sum method may lead to results away from the global optimum. The epsilon-method, which provides much better organization on the objectives and constraints, seemed a better method to select suitable members during the optimization process.

To shorten a long time of cyclic simulation for calculating the fatigue life of the structure, epsilon-method was implemented in two stages. In the first stage, weight, strength, cost and maximum of Mises and Tsai-Wu stresses were assessed. The evaluation of the second stage was carried out only on structures which have satisfied the provisions of the first stage.

Figure 4.14 shows flowchart of the proposed method used in that study to optimize the size and material combination in multi-material structures under two types of multi-axial static and fatigue loading.
The purpose of the study was to provide a comprehensive model with the ability of using any number and type of objectives and constraints. Therefore, the weighted sum method was not appropriate due to heavy reliance on the accuracy of the results to allocate an appropriate value of weighted coefficients and because it lacks the ability to detect the optimal solutions in non-convex regions. In contrast, The epsilon-method requires the designer to choose only one of the objectives as main objective and other objectives must be regarded as constraints. Generally, there are at least two main objectives in structural optimization problems, so that none of them can be preferred over the other or ignored. Designers often tend to follow the optimal variables in terms of improved multi-objective problems concurrently during the optimization.

Hence, integration of the epsilon-method and NSGA-II algorithm to simultaneously handle at least two objectives and some constraints of the problem was proposed and implemented in this paper (Figure 4.14). The epsilon-method gradually removes members that do not meet design constraints by assigning dynamic coefficients during the optimization process, and the NSGA-II method provides the possibility of handling at least two significant objectives simultaneously. The choice for the NSGA-II method was not only made because of considering two objectives, but also because of increasing the speed of the solution and convergence compared with genetic algorithm. NSGA-II prevented the premature convergence and being stuck in local optima due to the diversification of population.

Dealing with the multi-discipline problem in such structures with different loadings does not appear feasible due to the increased time demand of computerized solutions. In addition, time-consuming analyses of fatigue in structures that did not satisfy the conditions of problem constraints even in static faster analysis causes the waste of CPU time. To overcome this weakness, the priority of structural analysis with various disciplines based on the necessary time for FEM-analysis were sorted from the shortest to the longest analysis (Figure 4.14). This means that static analyses, that require less time, are carried out first and the percentage of members that have satisfied the primary constraints were passed on to the time-consuming fatigue tests by the applying epsilon-method.

The structure shown in Figure 4.15 is entirely made of steel and calls for a design with a lighter structure using lightweight materials without reducing the static and fatigue strength and without a significant increase in its cost. Dimensional variables of the structure for the cross section of the structure can be exchanged within the limits specified in Table of Figure 4.14, also in respect of the aesthetical point of view or connection with other parts in the assembly.
The possibility of replacing the middle section of the structure with composite layers was investigated instead of using steel (as a variable of material combination, Figure 4.15). If the value of “L” equals zero, it means there is no composite material in the structure, whereas a value of 760 is equal to the maximum allowable length of the second material.

The presence or absence of a composite part, causes a change of sudden behaviour in the profile structure and is a type of discrete variable. The composite part is made of eight unidirectional layers with symmetrical combination and a total thickness of 2 mm. The angle of each layer can be changed between 0 and 180 degrees. Two types of multi-axial loading are individually applied, static and cyclic loading, see to Figure 4.17.
In order to generate the different variations of structures, a random selection was executed from the design space using MATLAB software. Generated structures were subject to a static analysis which was performed in ABAQUS software. Results of the analysis for each structure including weights, angular rotation around the X- and Y-axes, cost, maximum von Mises stress in metal parts and maximum Tsai-Wu stress in composite parts were stored on a result matrix. It should be mentioned that angular rotation is used as a criterion to determine the structure strength.

After statistical analysis, the present population is evaluated with Epsilon method and some members with lower cost, Mises, and Tsai-Wu were sent to fatigue analysis in the next step (here weight and strength are mentioned as objectives). Coefficients of accepting the constraint values can be changed from a small value to a larger value during the iterations, so that at least half of the population always has the chance to attend at a next step. Fatigue analysis took about 10 times longer than the static analysis for this structure, so that a considerable saving in the total time of optimization was achieved by analysis of half population that did not satisfy the constraints of static analysis.

Fatigue failure criteria were considered only for composite parts, and structure failures due to fatigue of metal parts and adhesive joints between metal and composite were overlooked. Such assumptions were only made to reduce the total time of the optimization process, otherwise the method of dealing with more disciplines is similar to existing algorithms for the analysis of composite fatigue. The fatigue life of the composite part was studied on the basis of the strain energy density method. To consider both the effects of positive and negative, an improved fatigue model of multi-layered composites, which has recently been presented by the author was used [70]. This model, by using a VUMAT subroutine in ABAQUS, software calculates the fatigue life of all sections of the composite part. The lowest fatigue life in the part sections is considered as fatigue endurance limit.

After calculating the fatigue life, the epsilon-method again was conducted on the present population. The severity of constraint coefficient increases in such a way that up to fifty percent of the current population that satisfy the terms of the fatigue life constraint were sent to the next stage of the algorithm as parents. Figure 4.18 and 4.19 show how structures, with more Mises stress or less fatigue life than the others, will be deleted at every stage of the present population.
As can be seen in the diagrams Figure 4.18 and 4.19, some members that have less weight and higher strength than others, i.e. closest to the center of the diagram, may be eliminated in the first and second stages of applying the epsilon-method. That is similar to the function of a penalty constraint in the fitness function on the weighted sum method.

The distribution of weight and rotational angle for 200 randomly selected structures is shown in Figure 4.20. As expected, a clear conflict was seen between weight and strength (rotational angle) of the structures. The same conflicts occurred for weight/cost and strength/cost and are excluded from the discussion for the sake of simplicity.

**Figure 4.18.** Normalized values of weight and rotation angle vs normalized Mises stress. The first level of the epsilon-constraint method rejects the structures with higher amount of Mises stress

**Figure 4.19.** Normalized values of weight and rotation angle vs normalized Fatigue life. The second level of the epsilon-constraint method rejects the structures with lower amount of fatigue life

**Figure 4.20.** Variation of normalized weight and rotational angle for 200 structures
Figure 4.21 shows reduction in weight and angular rotation (increase in strength) of a structure during the iterations for the two-objective algorithm, in which the weight and strength were chosen as the main objectives by the NSGA-II method at the same time. A visualization of the optimized structure after nine steps of optimization is depicted above the Diagram. Although the composite materials are lighter and more stronger than steel, the algorithm has not replaced the complete middle section with composite materials because their cost are about five times higher than the steel. As the cost of structure rises with the length of the composite part, the defined constraint for the cost in the epsilon-method has removed structures that are too expensive.

**Figure 4.21.** Reduction of weight and rotational angle through the optimization loop (two-objective problem)
4.3.1 Realization of concepts

The categorization of concepts is the starting point for most structural optimization problems. Concepts are a collection of engineering ideas, which help to enhance the performance of structures. Obviously, the weaknesses of a considered structure must be listed at the beginning of optimization process and relevant concepts should be selected based on these weaknesses. Evidently, three main group of factors can be regarded as the most significant reasons for optimization of structures during the recent years [11, 189, 190, 191, 192, 262, and 263].

1- Tightening of environmental regulations towards reducing CO$_2$ emissions
2- Increasing safety regulations
3- Reduction of manufacturing cost to compete on the market

![Figure 4.22](image)

**Figure 4.22.** Necessity of reducing CO$_2$ emissions, increasing automotive safety, and reducing manufacturing cost are the main motivations for optimization of mechanical structures.

Various types of concepts have been recently introduced to satisfy the mentioned requirements. Furthermore, innovative technologies have been developed frequently. Here, not all of them will be presented. Instead this work will focus on the implementation methods to use some of the solutions in structural optimization. Here, one of the most comprehensive classifications of structural optimization concepts is illustrated [1], see Figure 4.23.

The first group of concepts, which has been classified in Figure 4.23, refers to the replacement of the type of part in the structure. The best known example, is the use of sandwich structures instead of sheet metal parts.

In the second group of concepts, the type of part remains the same, but the material of the part is replaced by lighter and stronger material. In other words, the material with the bigger ratio of stiffness/weight has better chance to be implemented into the new structure.

The third concept of lightweight design is not new solution in automotive industry. For a long period of time altering of part geometries has been considered as the simplest and cheapest way to enhance the structures, because there is no need to pay for new expensive material or changing the production setting. The part become stronger only with small modifications of current tools.
Altering of the part geometry is not independent from changing the part material. Structural design will be better optimized when geometry and material optimization are considered at the same time.

Since this study aims at proposing an optimization method for shell structures, concepts three and four (Figure 4.23) will not be considered for the possible concepts in the coming chapters.

However, topology optimization has been recently used to optimize the shell structures [219] but it commonly employed for casting and injection parts.

Since using the multi-material concepts in automotive bodies has been widely recommended in recent years, the author has added the last row to Figure 4.23 himself. That means, the connection type of parts in an assembly is considered as an efficient solution to enhance the structural performance.

Up to now, some of the most suitable methods were introduced, which were used to improve the structural performance in automotive bodies. They are not final concepts, they are improved very frequently or new ideas are born. One of the main objectives of this study is to

![Figure 4.23. Variety of design concepts in structural optimization [1]](image-url)
show the circumstances of implementation of the concepts into the optimization loop. Here it is called “realization of concepts”. In fact, this chapter of the study explains how the ideas and initial concepts of designers can be converted to a virtual structure. In other words, the proposed method of this studies able to automatically produce every concept with every variation in a virtual environment. This methodology does not only provide the reliability of producing all ideas, but also generates all possible combinations of ideas for designers.

**Figure 4.24.** A generic profile of an automotive body, consisting of inner and outer part with possible varieties of geometries and materials

Figure 4.24 can be taken as an example for the improvement of a structure using every possible geometry and material. The above-mentioned method (realization of concepts) enables all possible combinations of dimensional and material variables for the designer.

After choosing a concept for the original structure, the designer has two options to find his optimum design (Figure 4.24). (Original structure means the base structure which going to be improved).
- If the number of generated structures with different geometries and material properties is not huge and the evaluation of structures in FEM-software is feasible in time, he can evaluate all of them. Then by using Design of Experiment (DOE) or approximation techniques, lighter and stronger structures can be determined, [31, 36, 43, and 264] (section 2.2.1).

- If the number of possibilities are too many and evaluation time is too long, the designer has to use direct search or hybrid methods (section 2.2.2 and 2.2.3). In case of nonlinear behavior of the structure like crash analysis, using direct search methods is recommended. Those methods, instead of evaluating all possible combinations (which is almost impossible in practice), a small number of the whole population is randomly selected. Then by using evolutionary or metaheuristic approaches, the optimum design will be found after some iterations.

**Figure 4.25.** Evaluated structures are stored in a “Result Matrix”. There are two main groups of approaches to find the optimum design from different variations of structures.

It should be mentioned that gradient based methods because of their limitation to find the optimum structure in multi-objective problems with a huge number of input variables, are not considered as an approach in this study. Most of structural optimizations in automotive bodies deal with different types of load cases and need to be optimized for a couple of objectives.
4.3.2 Evaluation of concepts

This study uses the finite-element-method (FEM) to evaluate the specifications of generated structures (FEM-based optimization). Geometrical complexities of automotive structures, specific material properties of parts, and their connections cannot be calculated with mathematical equations. As different concepts for the improvement of structures were presented in the previous chapter, there is a wide range of criteria to evaluate the structures. This study is not going to introduce all required criteria for the evaluation of structures. Some well-known requirements of the automotive industry, which may be called “objectives in automotive structural optimization”, have been shown and listed in Figure 4.26.

- Improvement of static bending and torsion stiffness
- Enhancing of NVH (noise, vibration, harshness) and fatigue performance, against internally excited and externally excited vibration
- Crashworthiness
- Improving of sub-assemblies against deviation of temperature
- Recyclability
- Reduction of \( CO_2 \) emission
- Reduction of manufacturing costs

**Figure 4.26.** Most recent common goals in the automotive industry
Designers can evaluate some objectives at the same time and obtain a unique value (e.g. summation of normalized objectives) as the fitness of structure [60, 64]. According to recent studies, it would be possible to convert all objectives into a single objective or taking into account the other objectives as constraints (chapter 2.3.2). Since the hybrid metaheuristics algorithm (chapter 3.2) of this study needs to select and exchange best members during the iterations, it has to use one value as the fitness of structure. In order to dedicate different priorities to various objectives, every objective gives a ratio in the first step of optimization.

**Table 4.3.** Weight, cost, bending, and torsion for some random configuration of variables from the presented structure in Figure 4.23.

<table>
<thead>
<tr>
<th>Structure #</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Material</th>
<th>Weight (Kg)</th>
<th>Cost (€)</th>
<th>Bending Stiffness (N/mm)</th>
<th>Torsional stiffness (N.m/deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>32</td>
<td>19</td>
<td>58</td>
<td>80</td>
<td>Laminate</td>
<td>1.2</td>
<td>12</td>
<td>18.5</td>
<td>5.3</td>
</tr>
<tr>
<td>#2</td>
<td>25</td>
<td>28</td>
<td>26</td>
<td>92</td>
<td>Steel</td>
<td>1.6</td>
<td>1.8</td>
<td>12</td>
<td>2.5</td>
</tr>
<tr>
<td>#3</td>
<td>48</td>
<td>34</td>
<td>34</td>
<td>65</td>
<td>Aluminium</td>
<td>1.5</td>
<td>3</td>
<td>8</td>
<td>1.7</td>
</tr>
<tr>
<td>#4</td>
<td>42</td>
<td>12</td>
<td>45</td>
<td>100</td>
<td>Steel</td>
<td>1.7</td>
<td>1.7</td>
<td>9</td>
<td>1.9</td>
</tr>
<tr>
<td>#5</td>
<td>35</td>
<td>38</td>
<td>55</td>
<td>84</td>
<td>SMC</td>
<td>1.3</td>
<td>10</td>
<td>10</td>
<td>2.1</td>
</tr>
</tbody>
</table>

In chapter 4.4.1, a B-pillar assembly under two load cases will be optimized. The optimal geometrical and material parameters will be obtained for highest stiffness, lowest weight, minimum cost, and some more factors, considering the manufacturing constraints. A specific approach to evaluate the production cost and manufacturing limitations will be presented in chapter 4.4. At the end of the evaluation phase, a table (such as Table 4.3) is generated which enable the designer to compare current objectives with expected objectives. In case that the expectations are satisfied, the optimization process will be finished. Nevertheless, new structures are produced which have inherited the properties of the last generation. Here, it would be useful to categorize the evaluation approach into two well-known groups:

1- Evaluation of all generated structures (all possible configurations)
2- Evaluation of a portion of all possibilities and using a surrogate model during the optimization iterations

If the structure shows nonlinear behavior under subjected loads such as crash situation, an instant model is recommended [30, 166, 253, and 265]. If there is not enough time for simulation, an instant model has been suggested as well. Of course, it is necessary to check the accuracy of the surrogate model with real FEM results [35, 253, 267, and 268]. Sometimes, in order to obtain an accurate estimation of the structure stiffness under high nonlinear loads in a relatively short time of optimization, designers prefer to implement quasi-static load cases [146]. Obviously, the reliability and accuracy of optimal design should be fine-tuned at the end of the optimization under dynamic load cases.
4.3.3 Optimization algorithm

In the last two chapters, it has been shown how designers use a concept from categorized concepts to generate the multi-configuration structures. In addition, it has been introduced, how the generated structures are evaluated and predefined objectives are obtained. At the end of these two steps, a matrix consisting of input variables and their related objectives will be constructed. However, following cases will easily mislead the optimization algorithms towards local optima instead of global optima.

- Because of discrete variables, structures show discontinuous behaviors.
- Existence or non-existence of a part in a structure causes discontinuous behaviors.
- Altering of boundary conditions during crash simulations causes nonlinearity as well.
- Failure of materials during dynamic simulations changes continuity of behavior.

A semi-optimal design instead of optimal configuration may cause a huge difference in the production costs in automotive industry. Because of the above-mentioned reasons, a strong and flexible optimization algorithm is required, which would be fast and as accurate as possible without being at risk of being stuck in local optima. Here, some key points of hybridization of PHMs algorithms are presented:

- As explicitly mentioned and assessed in chapter 3, there is no algorithm, which has entirely all specifications. Consequently, a combination of good characters of some algorithms seems a good idea to overcome the weaknesses of a single algorithm. Since population-based metaheuristics (PMHs) have a random search behavior, it would be possible that they start with a poor population or their operators do not improve members as expected. Then, the idea of team working of some PMHs with different operators could remove the mentioned weakness [54]. The chances of three algorithm to find the global optimum should be better than the chance of one algorithm.

- If one algorithm in a group has not enough agility or chance to find fitter members, it is not an good idea to immediately eliminate it from the search group. By adding some good members from other groups, it gets the opportunity to show its abilities.

- If some algorithms with a good starting point (single-solution based, like metaheuristics simulated annealing and Tabu search) are quick in finding better neighbors, they should be supported from PMHs algorithms to overcome the weakness of a bad starting point.

- Weaknesses of PMHs in their slow local search ability can be improved by using SMHs.
If the whole hybrid algorithm shows a slow convergence rate or runs the risk of being stuck in local optima, it should adaptively update the initial parameters to change its currently wrong strategy.

The combination of the mentioned factors and considering their interaction will design an intelligent algorithm with a high reliability and in reasonable time for structural optimization problems [48, 55, 109, and 179].

In the next chapter, three main milestones, which were introduced for the optimization process (Figure 4.1), will be implemented to solve a real structural optimization problem. It will be shown how the initial ideas of designers are generated and evaluated and how the strong members even become stronger towards finding the optimum stricter through optimization iterations.
4.4 Multidisciplinary optimization of a B-pillar structure

4.4.1 Problem definition

The design problem is aimed at reducing the weight and increase the strength of a B-pillar structure while keeping the cost of the structure less than the predefined target. The B-pillar has to bear many static and dynamic loads during the normal riding of a car on/off-roads or in accidents. In this study, two types of load cases have been considered to evaluate the B-pillar structure. The first load case is a static load, which is a simulation model of a rollover test (Figure 4.27), and implements a force with some relations of the car weight onto the roof of the car in three directions, Figure 4.28.

Figure 4.27 Importance of B-pillar strength to ensure the survival of occupants in a rollover crash [source: commuteronline.com]

Figure 4.28. FMVSS No. 216a, roof crush resistance test procedure [61]
The second load case is a dynamic load, which simulates the side crash (Figure 4.29) with a moving barrier as one of the most significant factors in the assessments of crashworthiness.

**Figure 4.29.** Importance of B-pillar strength to ensure the survival of occupants in a side crash [source: iihs.org]

The first reason for choosing two different load cases as the point of static and dynamic load is to demonstrate the ability of the proposed optimization tool under more than one load case. The second reason is to illustrate two different approaches to deal with structures under dynamic (nonlinear behavior) loads. It is necessary to mention that in this study, instead of evaluating the whole vehicle against the crash impulse, only the structure of B-pillar, consisting of an inner and an outer part, has been investigated (Figure 4.31). The reason for this simplification is that this study tries to introduce a comprehensive approach to find the optimum solution out of a wide range of concepts. Therefore, the size of the model should not play a significant role in the whole process. On the other hand, simulation for whole vehicle would be

![Figure 4.30. Side crash definitions based on European regulation of UN R95 [61]](image)
a very time consuming process and seems an unfeasible task. However, the proposed tool in this study which will be introduced in the next chapters, can be used for any subassemblies of the car body and even for the whole vehicle with any number of continuous or discrete variables and load cases. In order to consider the effects of eliminated parts which connect the B-pillar with the car body, alternative elements like springs, dampers, and center of mass have been replaced in the simulation model.

**Figure 4.31.** View-cut of B-pillar, which is optimized in this study

In this way, the behavior of the B-pillar model was as close as possible to real structure in the car body. Detailed information to choose replacement elements is discussed in chapter 4.4.3.

**Table 4.4.** Objectives and constraints for optimization of B-pillar structure in this study.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Baseline* Value</th>
<th>Objective/Constraint</th>
<th>Load case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (Kg)</td>
<td>20.31</td>
<td>&lt; 80% Baseline</td>
<td>No load</td>
</tr>
<tr>
<td>Total cost** (€)</td>
<td>24.37</td>
<td>&lt; 150% Baseline</td>
<td>No load</td>
</tr>
<tr>
<td>Top-Intrusion (mm)</td>
<td>118.8</td>
<td>&lt; 127 mm</td>
<td>Roof crush resistance, FMVSS No. 216a</td>
</tr>
<tr>
<td>Intrusion B-Pillar to driver seat (mm)</td>
<td>130</td>
<td>&gt; 125 mm</td>
<td>Side crash, UN R95, 50 Km/h 90°</td>
</tr>
<tr>
<td>Energy absorption (N.m)</td>
<td>450</td>
<td>Maximize</td>
<td>Side crash, UN R95, 50 Km/h 90°</td>
</tr>
<tr>
<td>Manufacturing restrictions***</td>
<td>No value</td>
<td>Minimize</td>
<td>No load</td>
</tr>
</tbody>
</table>

* Specification of original B-pillar; consists of one inner and one outer part made of steel sheets.
** Cost of raw material plus cost of production, consists of machines, tools, and joinings.
*** Complexities of sheet metal forming.
The first column shows the specifications of design objective and constraints, the other columns show the detailed information of the mentioned features. Again, for simplification, small features consisting of small chamfers, fillets, holes, deformations have been eliminated. In addition, the supplementary reinforcement brackets inside the B-pillar structure located in the top anchorage point of the seat belt and the door lock were not been considered in this study. Therefore, only two simple steel parts from the original B-pillar, which are going to be optimized in this study.

**Figure 4.32.** Original B-pillar structure made of two steel parts for inner and outer

The structure weight is calculated by multiplying volume with density, which is obtained from 3D-model. In the original model, this weight was 20.31 kg, which is written down as baseline weight in Table 4.4. The optimized structure was supposed to be 20% lighter than the original one, that means 80% of baseline weight. During the optimization, process structures with a weight of more than 80% of the baseline weight have less chances to be selected as the optimum structure at the end. The second specification for the structure in the second row of Table 4.4 is the total cost of structure. In this study cost of structure, does not only consist of raw material cost, but also of production cost, tooling cost, and joining cost (Figure 4.33).

**Figure 4.33.** The cost of every structure is calculated by summation of these items
Although more subjects related to the final cost calculation could be determined only the mentioned items have been considered in this study. Its aim is to demonstrate the circumstances of considering the production cost as an important objective in the optimization process. Any additional items to calculate the cost of structure could simply be added.

**Raw material cost**

As it will be illustrated in chapter 4.4.2, some different groups of structures with various raw materials and part combinations are suggested as the candidates to participate in the optimization process. Therefore, they have a different weight and consequently different cost. A wide range of metal, plastic and composite materials has been selected to make the inner and outer part. Table 4.6 and 4.7 show the physical and mechanical properties of candidate material in this study.

**Production cost**

The number of machines to produce every part of the structure is considered as a metric to compare the production cost of different structures. That means, instead of using the real cost of machines the number of machines determines the production cost. Based on this assumption, there is no difference between different tonnages of press machines, for sheet metal parts and also injection machines for plastic parts and RTM facilities. If the outer part of a B-pillar is made of a unique part with a transfer molding hydraulic press (Figure 4.34), for example, then the production cost will be mentioned as one. If the outer part is made of two individual parts, it will need two press machines to produce Figure 4.35.

![Figure 4.34](image.png)

**Figure 4.34**. Press line machine to produce a single-piece outer part

Although it may be possible to design special tools or increase the shift time of one machine to produce two parts with only one machine such solutions mentioned tricks and solutions have been not considered in this study. The main idea of calculating the production cost is to make a difference between the structures with a higher quantity of parts with simple structures. By this way, a structure with two parts should be cheaper than a structure with three or four parts,
even if they may have a similar weight. However, if machines cost are available, it will be possible to use the real price of machines instead of their quantity in the used approach of this study. The number or price of machines will be normalized based on the number or cost of the most part structures.

Figure 4.35. Press line machine to produce a two-piece outer part

Table 4.5. Calculation method of production cost for some structures with a different number of parts. (The same approach is used to calculate the tooling and joining cost.)

<table>
<thead>
<tr>
<th>Number of parts in current structure</th>
<th>Minimum number of parts* / Number of parts in current structure</th>
<th>Normalization</th>
<th>Production cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 / 2</td>
<td>1 - (2 / 2)</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>2 / 3</td>
<td>1 - (2 / 3)</td>
<td>0.33</td>
</tr>
<tr>
<td>4</td>
<td>2 / 4</td>
<td>1 - (2 / 4)</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>2 / 5</td>
<td>1 - (2 / 5)</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>2 / 6</td>
<td>1 - (2 / 6)</td>
<td>0.67</td>
</tr>
</tbody>
</table>

* A baseline structure consisting of one inner and one outer part has the minimum number of parts in this study, (that is 2)
Tooling cost

The number of dies to produce all parts of every structure is considered as the criterion for tooling cost in this study. For instance, if the outer part of the B-pillar is made of two parts, it will need two dies to produce it, see Figure 4.35. The background idea of regarding tooling cost is to consider the increase of cost for structures with more parts. In two-piece structures, stronger material is applied to the right location and is expected to show better performance than the unique part. However, the cost increasing due to more tooling necessities should be considered as well. As this study differentiates between more and less part structures regarding the tooling cost, using the number instead of the real price for tooling is acceptable. However, as explained in the production cost section, it will be already possible to use the real price of tools without reconstructing the optimization algorithm process.

Joining cost

In multiple-part structures there is one joining process like welding, gluing, screws, etc. for each two parts. Consequently, a higher number of parts causes a higher number of connections and more joining costs. Figure 4.36.b shows that for the production a B-pillar made

![Figure 4.36.a](image1.png) B-pillar made of a single outer piece needs one welding unit

![Figure 4.36.b](image2.png) B-pillar made of two outer pieces needs two welding units
of two-piece outer parts, one more joining station should be utilized compared to a B-pillar made of one single piece (Figure 4.36.a).

However, the production line and welding fixtures may be designed in such a way that a three-piece structure might be weld with one welding unit and a related robot. Hence, it would not be cheaper or simpler than the one welding unite. With the mentioned hypothesis, in this study all two-piece structures will take the number one as the joining cost and all three-piece structure will take two as the joining cost and so on, see Table 4.5. Finally, the joining cost of every structure with every number of joining location will be normalized base on the most joined structure.

Different mechanical properties should be considered in field of joining. Because of residual stresses and different material properties, the strength of a two-piece part is different from a one-piece part. In this study, an individual parameter as a constraint has not been defined in Table 4.4 to evaluate the quality of more joined structures. If the structural quality due to joining decreases or increases, it will be seen during the FEM of the structure under dynamical or statistical loads. Here, for simplification, no specific interaction properties have been considered for joining. Regardless of the joining techniques (welding, cohesion, screwing, fusion, etc.) all connections have similar properties and no damage failures. However, it will be possible to define the initiation and evolution properties for connections and optimize the joining properties by dimensional and material optimization. As explained earlier, because of prolongation of the input variable length, the effect of joining properties is neglected in this study and joining are considered to have effects on the fitness of the whole structure.

As a predefined target, the optimum structure should not be 1.5 times more expensive than the baseline structure (Table 4.4, second row). The reason for accepting this extra cost is that making stronger and lighter structures without using lightweight material is almost impossible. These kinds of material are more expensive than the conventional material [194].
Top-intrusion

The third row of Table 4.4 shows the displacement of the force point at the top of the B-pillar in a roof resistance test based on FMVSS No. 216a [61]. According to the acceptance levels of this regulation, the maximum deformation of the B-pillar towards the occupant compartment must be less than 127 mm. In order to consider the effect of roof and side reinforcements, which were removed for simplification, two series of spring elements in X and Y direction were used, see Figure 4.37.

\[ \text{FORCE = (GWWR(Kg) \times 9.81 m/s^2 \times 3) \text{ N}} \]
Where: GWWR = Gross Vehicle Weight Rate = 1,594 Kg

With respect to the force direction:
- \( F_1 = 3,698 \) N
- \( F_2 = 19,979 \) N
- \( F_3 = 42,322 \) N

**Figure 4.37.** Three times GVWR of a middle range passenger car is implemented to the top point of the B-pillar in three direction. In upper boundary, effect of roof cross members and side assembly have been replaced by two springs in X and Y direction. In lower boundary, inner and outer parts have been fixed in all directions.

The amount of intrusion for the baseline structure and the property of replaced elements was shown in Figure 4.56. The coefficient ratio of springs were chosen in such a way that the deformation of the B-pillar for the baseline structure (before optimization) is equal to 70% of acceptance level (Table 4.4, third row). However, this amount may not exactly model the behavior of the real structure, but in the case of a comparative study, they should be acceptable.

In chapter 4.4.3, more detailed information about FMVSS modelling and related investigation will be introduced. In this study, an intrusion of 108 mm, which is safer than 127 mm, has been considered to evaluate the performance of the B-pillar structure against a roof crush load.
Intrusion of B-pillar to driver seat

The fourth specification of the B-pillar, which must be evaluated, is the distance between an imaginary point at the top of the B-pillar on the inner part to the driver's head after side collision according to UN R95 [61]. Since no dummy participates in the simulation, the displacement of the B-pillar is measured from a predefined point, which moves along the structure during the side impulse. In order to measure the effect of the vehicle weight and the resistance of connected parts to the B-pillar, which have been neglected so far, two center of masses were mounted, see Figure 4.38.

![Figure 4.38](image)

*Figure 4.38. Vehicle mass and resistance of car body against side crash, are replaced by mass centers at the bottom and top regions.*

These two centers of masses allow the structure to move in Y direction with equivalent inertia due to the vehicle weight [146]. After a real side crash, the distance of the inner part to the mentioned imaginary point must be bigger than 125 mm. It will be shown in chapter 4.4.5 that the baseline structure made of steel shows a 115 mm deformation after the side impactor pulse. This amount has been considered as the allowable intrusion of the B-pillar in this study.

The optimized structure should satisfy the requirements of the R95 regulation. However, the single B-pillar with equivalent mass centers may not precisely work as it would be in a real vehicle. Therefore, instead of a 125 mm intrusion value, 130 mm, which was obtained from the baseline simulation, will be considered as the acceptance level in this study. Of course, this amount leads more conservation than the suggested value in the R95 regulation.

Energy absorption

The fifth specification of the B-pillar, which must be fulfilled by the new structure, is the energy absorption due to the side crash. In the R95 regulation, no specific value has been defined for this absorption and obviously, it should be as high as possible. That means that the B-pillar
structure should tolerate a high force for a long period of time. Figure 4.39 shows a simple schematic the of B-pillar structure with a high ability of energy absorption.

In an ideal case, the structure in Figure 4.39 with the allowable amount of distortion could absorb a high volume of energy and transfer a minimum rate of acceleration to occupants. However, such structure seems not to be achievable in a car body, at least on mass production level. Alternatively, it would be possible to modify the geometry and material property of the B-pillar cross-section in such a way that it comes closer to the requirements [143, 144, and 145].

Figure 4.39. An ideal concept of B-pillar, should absorb more force from impactor with less deformation which provides more survival space and consequently less transferred acceleration to occupants [147, 189]

In chapter 4.4.2 and 4.4.3, some similar and applicable ideas will be introduced and evaluated, and finally in chapter 4.4.6 the result of the assessments will be summarized and compared. In the current study, the maximization of energy absorption for B-pillar structure has been planned and the absorbed energy of the baseline structure obtained as 450 N.m from FEM results, see Table 4.4, fifth row.

Manufacturing restrictions

The last row of Table 4.4 presents not an objective but a limitation for the optimization of the B-pillar. The formability of high strength steels has been considered as a manufacturing limitation in this study. Before introducing the circumstances of restriction implementation, a brief survey of some manufacturing limitations in car body parts made of sheets (not casting or injection parts) will be presented in chapter 4.4.4. The discussion of manufacturing limits, which has been introduced in this study, needs more details about the used materials and their permissibly in this study. Therefore, after introducing the input variables, which consist of dimensions, materials, etc. in chapter 4.4.2 a detailed description of manufacturing restrictions will be presented in chapter 4.4.5.
4.4.2 Realization of structures from input variables using CAD

In chapter 4.3.1, some essential motivations for the optimization of mechanical structures were introduced. Along with that, some command approaches were categorized in Figure 4.2 [1]. In this chapter, some well known and of course applicable solutions in lightweight industry will be introduced to increase the strength of the B-pillar against roof and side crash loads as illustrated in Figures 4.27 and 4.29. The selection of solutions in this study, tend towards using different materials in one structure, namely multi-material structures or hybridization. Therefore, the types of combination for different kinds of material are the main challenge. The presented methods in this study are not the complete and final solutions to enhance the performance of a structure. But this study aims at showing how the different ideas could be examined and the optimum solution could be found. Every number and type of ideas could be added to current ideas and is evaluated and optimized according to the proposed method of this study. This study has suggested five ideas or concepts to increase the performance of a B-pillar under the given objectives and constraints in Table 4.4. Each concept will be presented by the number or type of used parts in the structure. Then the applied variable will be listed. Variations of every variable between minimum and maximum levels and the combination of variables offer a wide range of structural design with different properties.
Concept 1: Two-part assembly

In this concept, different dimensions of cross-sections are generated in order to increase the strength of the original B-pillar structure. At the same time, different materials will be assigned to the inner and the outer part. Figure 4.40 shows the result of variation for eight dimensional variables to generate different profiles of the B-pillar structure.

![Figure 4.40](image)

**Figure 4.40.** Changing each parameter or variable between its minimum and maximum level generates different cross-sections. The optimum cross section should be between these levels.

Simultaneously, every part can consist of different predefined materials. In this study, four materials have been considered for parts made of sheet material. This quantity of materials could be increased by any number. Table 4.6 shows mechanical properties of sheet materials that are employed in this study.

**Table 4.6.** Mechanical properties of nominated materials for inner and outer parts

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (Kg/mm²)</th>
<th>E Module (Gpa)</th>
<th>“YS” (Mpa)</th>
<th>“UTS” (Mpa)</th>
<th>A (%)</th>
<th>Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel Ductibor® 500</td>
<td>7.80E-06</td>
<td>210</td>
<td>400</td>
<td>550</td>
<td>≥16</td>
<td>Arcelomittal [271]</td>
</tr>
<tr>
<td>Steel tribond® 1400</td>
<td>7.80E-06</td>
<td>210</td>
<td>800</td>
<td>1300</td>
<td>&gt;5</td>
<td>Thyssenkrupp [272]</td>
</tr>
<tr>
<td>Aluminum EN AW 6082</td>
<td>2.70E-06</td>
<td>70</td>
<td>240-260</td>
<td>295-310</td>
<td>7.5</td>
<td>Gleich Aluminium [273]</td>
</tr>
<tr>
<td>CFRP TepeX® dynalite 201</td>
<td>1.46E-06</td>
<td>55</td>
<td>700</td>
<td>700</td>
<td>1.5</td>
<td>LANXESS [274]</td>
</tr>
</tbody>
</table>
Different types of material for considering the strength/weight and strength/price point of view were intentionally selected here.

- Ductibor® is a common steel sheet in the automotive industry. It has a valuable price, good elongation for formability, and the medium mechanical properties.
- tribond® is an especially steel material, twice as expensive as the usual steel sheets, low elongation and difficult formability, and high strength in mechanical properties.
- Aluminum EN AW 6082 is a common aluminum sheet in the automotive industry, with a relatively good formability, about three times lighter than steel, a valuable price, and almost medium mechanical properties (Table 4.9).
- CFRP Tepex® is a new material, at least four times expensive than steel sheets, five times lighter than steel, good formability with some considerations, and higher mechanical properties than usual steels.

Composite sheets, can achieve higher mechanical properties by using different stacking sequence variations. The author [239] has already assessed the possibility of stacking sequence optimization of a composite part for the optimization of an A-pillar structure. That means that the inner and outer parts could be made of unidirectional or woven laminate (Figure 4.40). Then, the optimum number, thickness, material, and orientation of every layer can be find through the optimization process. This was ignored in this study to avoid very long input variables length. Figure 4.41 shows a randomly generated identification (variables vector) to make the B-pillar with two pieces (inner and outer) and materials.

![Diagram](image)

**Figure 4.41.** Randomly generated identification for two-piece assembly. The first eight numbers identify the dimensional variables and the four last numbers determine material and thickness of inner and outer parts.
This row of variables, which is called Part ID, generates different structures in CAD. A visual basic script updates the parametric 3D model of the B-pillar structure in CATIA [149]. Parameters to vary the cross-sections of the B-pillar were shown in Figure 4.37. Updated geometries will be automatically saved in IGS format and produces as many as different structures as given number. The IGS files will be sent automatically to ABAQUS [82] for a meshing process and the number of elements and nodes coordination will be sent back to the optimization tool. Based on the desired preciseness, the engineer can define the size and type of elements. In next section, an input file of the simulation (*.inp format) for FEM-analysis will be made based on the test requirements (in this study roof crash and side crash). This input file consists of elements and nodes for each part which have been generated in the last step as well as materials, connection between the parts, and load cases. There is no limitation to the number of any type of analysis, and any kind of desired examination, like structure under fatigue load or temperature variation could be added to existing analyses. FEM-analyses of the structure are automatically carried out and the results in the output format of ABAQUS (*.odb) will be stored. Figure 4.42, shows the flowchart of generating structures from Part ID to the simulation step.

![Figure 4.42](image.png)

The number of structures, which can be produced with different cross-sections and material combinations for inner and outer part, are estimated as $1.024 \times 10^9$.

The optimization algorithm in this study starts with an initial population consisting of $n$ randomly generated structures and after a few iterations finds the optimum two-piece structure, which satisfies all objectives and constraints, listed in Table 4.6. The circumstances for creating the fitness function for the multi-objective optimization of this study will be discussed in chapter 4.4.6.
**Concept 2: Three-part assembly**

The second idea for increasing the strength of the B-pillar is using lighter and stronger materials only in the necessary sections. Determining of necessary sections is automatically carried out by the optimization algorithm. Lighter/stronger materials with tailored length will be assigned to the ideal location of the part. As a less than ideal concept, it could be possible to make the complete outer part out of high strength steel or composite laminate without an optimization process. But this approach may not be feasible for many reasons. There is no guarantee that all objectives listed in Table 4.4 like weight, cost roof resistance, and energy absorption of side crash can be fulfilled at the same time. Especially the influence of cost due to using high amounts of light/strong materials will very likely be criticized. Therefore, despite the easiness of using light/strong materials in the whole outer part, this could not be applicable. A suitable alternative is the combination of light/strong materials and traditional materials for the outer part. Like in the two-part assembly, variations of cross-sections like height, width, and wall angles are considered as dimensional variables in this kind of assembly (Figure 4.43). The length of the outer part is divided into two sections and two equal or different materials are assigned to each section.

![Figure 4.43](image)

**Figure 4.43.** Length of outer part is divided into two sections and two equal or different materials are assigned to each section.

The estimation of dividing distance is not easy in case that different load cases and variables. The determination of the length also depends on the cross-section and material of the section at the same time. Thus, a wide range of dimensional and material variations must be generated and evaluated to find the optimum design. For this kind of structure an identification code (assembly-ID) is created. By altering this ID, different specifications of the structure will be generated and can be evaluated (Figure 4.44).
This combination of geometry, material, and length of parts enables twice as many structures as the first group and probably more enhancement for the performance of the B-pillar. In the generated structure combinations, two dissimilar parts may be located beside each other. In this study, 25-35 mm overlap length has been suggested for connecting parts together without paying attention to the material of the neighboring part. In fact, the joining of these parts could be one of the following types: spot welding, line welding, projection welding, sim welding, laser welding, cohesive bounds, fusion, screwing, and riveting or a combination of some of them. As a comparative study, section 4.4.3 shows the difference in the structural performance with and without joining damage. However, the proposed tools of this study could offer the opportunity of adding the specifications of the joining methods as input variables and optimize them, but for simplification, it has been ignored here.
Concept 3: Four-part assembly

The optimization of every structure is normally carried out during design process (before starting the tooling design). The results of the optimization can be used to make millions of parts for a couple of years. Therefore, it is worth that the optimization of the structure is done as precise as possible for every imaginable idea and concept. In this section, the outer part of the B-pillar is divided into three lengths to provide the opportunity of using three different materials with different thicknesses (Figure 4.45).

![Figure 4.45. More flexibility for material combination through multi-sections](image)

Similar to the first and second groups, cross-sections are able to variate accordingly. This combination of geometries, thicknesses, lengths, and material properties give three times as many opportunities for structural constructions as the first group and is twice as many as the second group. An ordinary identification code for a four-piece structure is shown in Figure 4.46.

![Figure 4.46. Randomly generated identification for four-piece assembly. The first eight numbers identify the dimensional variables and the six last numbers determine material and thickness of inner, lower outer, middle outer, and upper outer part](image)
Concept 4: Bead foam reinforced assembly

The outer part of the B-pillar structure has experienced its maximum improvement by being divided into two and three different longitudinal properties. It seems that enhancing the mechanical properties has reached its limits after dimensional and material variations of the outer part. A new idea, which is introduced and added to previous concepts, is using auxiliary parts to improve of the strength of structure. These auxiliary parts could be made of sheets, plastics or cellular raw material. The idea behind it is to increase the absorption properties in parallel to decrease the intrusion of the impactor in a side crash. With this motivation, this section introduces some cellular materials that can be incorporated between the inner and outer part of the B-pillar. Similar to the previous groups, dimensions of cross-sections and related thicknesses as well as the material of inner and outer parts can be varied. For cellular part some varieties were considered. First variation is the length and position of the filler material along the structure (Figure 4.47).

![Figure 4.47](image)

**Figure 4.47.** Different lengths and positions of filler material caused different performances of the structure against excessive deformation of the outer part

The second variation of filler materials is to use two different type of materials, see Table 4.7

**Table 4.7.** Mechanical properties of filler materials (metal and polymer based foams)

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (Kg/mm³)</th>
<th>E Module (Gpa)</th>
<th>&quot;YS*&quot; (Mpa)</th>
<th>&quot;UTS*&quot; (Mpa)</th>
<th>A (%)</th>
<th>Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum Foam, Forminal®</td>
<td>6.00E-07</td>
<td>6.6</td>
<td>25</td>
<td>40</td>
<td>no</td>
<td>Fraunhofer [275]</td>
</tr>
<tr>
<td>Expanded PP, Neopenol® 100</td>
<td>1.00E-07</td>
<td>0.013</td>
<td>0.45</td>
<td>0.55</td>
<td>no</td>
<td>BASF [276]</td>
</tr>
</tbody>
</table>

* Drucker-Prager yield criteria have been used here
Results of finite element analyses show that filler material prevent extensive deformations of outer parts after collision. In fact, they reduce the degradation rate of failure energy and therefore improve the structural performance compared to a non-filled structure.

**Figure 4.48.** Al-foam, manufactured by filling air or other gases into molten aluminium composites [282].

**Figure 4.49.** Expanded Polypropylene (EPP) bead foam, manufactured by sintering the raw material using steam flow [284].

Some related researches in field of crashworthiness of foam-filled structure are introduced in [100, 283, 285, and 286].

An example of an assembly ID that is employed to create structures with filling materials is demonstrated in Figure 4.50.

**Figure 4.50.** Randomly generated identification for foam filled assembly. The first eight numbers identify the dimensional variables and the seven last numbers determine position and length of filler part, material and thickness of inner, upper, filler, and outer part.

Using these materials definitely leads to increase the weight and cost of the structure. However, it brings relatively good enhancement for other predefined requirements listed in Table 4.4. The overall improvement of the structural performance of this group compared to other presented groups will be discussed in chapter 4.4.6.
**Concept 5: Ribs reinforced assembly**

The idea of using filler material between the inner and outer part has been examined by using polymer ribs in this section. These ribs could be injected when the outer part has been inserted to the injection mold. In addition to dimensional variables and materials of inner and outer parts, some dimensional and material variables have been defined for ribs construction which allows them to be located everywhere along the length with different resolutions (Figure 4.51).

![Diagram of ribs](image)

**Figure 4.51.** Variation of length and position of ribs as a filler material for B-pillar

Two types of polymer materials have been selected for ribs in this study.

**Table 4.8.** Mechanical properties of rib parts as filler material

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (Kg/mm³)</th>
<th>E Module (Gpa)</th>
<th>~YS* (Mpa)</th>
<th>~UTS* (Mpa)</th>
<th>A (%)</th>
<th>Supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABS 530/3.1 CF</td>
<td>1.08E-07</td>
<td>6.9</td>
<td>no</td>
<td>65</td>
<td>1.2</td>
<td>ALBIS [277]</td>
</tr>
<tr>
<td>PA6</td>
<td>1.59E-07</td>
<td>10</td>
<td>no</td>
<td>65</td>
<td>1.5</td>
<td>PENTAGON [278]</td>
</tr>
</tbody>
</table>

* Hyperelastic model has been used here

The connection of the ribs with the outer part has been ideally assumed without any damage criteria in this study (note: outer part could be randomly made of aluminum, steel or composite lamina). The characterization and optimization of this connection, which plays an essential role in the structural performance, is a major challenge of some research centers nowadays. However, for simplification, it has not been considered in this study.
Some other possible concepts

The outer part of the B-pillar structure with a weight of 8-12 kg and length of 1200 mm still has the potential to be examined with regards to different sections, materials, and extra partitioning. Recent developments and research on B-pillar optimization are the best evidences.

Figure 4.52. Other improvement approaches beside deviation of geometry and material
Sheet metal or composite part have the possibility of becoming stiffer using local deformation (stiffeners) or fiber reinforcement composites. These type of improvements with or without extremely increasing the overall weight enhance the strength of the structure in desired locations. Figure 4.53 shows a schematic description of a local deformation and fiber reinforcement in the outer part of B-pillar, which may leads to more bending stiffness in side crash.

Figure 4.53. Inserting longitudinal deformation (left) and applying fiber patch as some other possible solutions to increase local stiffness of B-pillar

The optimum location of the deformation and its width, height, and corner radiuses needs to be inserted precisely using optimization tools. Some commercial software like TOSCA has the possibility of bead optimization [16]. Fiebig [86] employed evolutionary algorithms to find the optimum patch and section of beads on a rectangular sheet metal part under concentrated force in center. The author [241] used fiber reinforcement patches to increase the bending and torsion stiffness of a sheet metal part under three axial load cases. Two types of evolutionary algorithm (GA and FA) were implemented to find the optimum length and trajectory of two patches along the part’s surface. Reinforcement of B-pillar using of deformation and patch fiber approach presented some possible solutions. They will not be employed in this study for simplification and can still be considered as future work.
Conclusion of proposed concepts

Five ideas were introduced to improve the performance of the B-pillar by reducing the weight and cost and increasing the mechanical strength of the B-pillar structure. These concepts are illustrated in Figure 4.54.

![Figure 4.54. Hybridization of B-pillar. Five concepts were considered for evaluation in this study with the aim of designing lighter and stronger structure.](image)

As explained earlier, these ideas are only some examples out of existing concepts for structural improvement. This study aimed at the structural optimization process, evaluating a realized concept and finding the best design. Some other ideas that might be added to existing concepts were shown in Figure 4.52. They could complete with the five mentioned ideas of this study to find the best configuration for the B-pillar structure.

The final output of the realization process as demonstrated in Figure 4.42, is an ASCI file consisting of all specifications of components in the assembly such as nodes coordination, materials, interactions, boundary conditions, loads, and so on. Since two types of load cases have been suggested for the optimization problem in this study, thus two ASCI files will be generated here (roof crush and side crash). Therefore, for five proposed concepts, ten ASCI files will be automatically generated at the end.

Furthermore, it is possible to combine the individual concepts to obtain more structural variety. For instants, the combination of a three part structure with an Aluminum foam-filled structure may give more flexibility to the designer for optimization results. But in this study, for simplification the combination of concepts has not been mentioned.
After determining concepts, their performance must be compared and the best one can be selected. Below, the optimization tool of this study is described as a flowchart; the detailed procedure will be described in the next sections. Two types of optimization behavior have been considered in this study. The first is using an exact FEM-model to evaluate the structures under static and linear behavior (Figure 4.55.a). The second is using a surrogate model instead of an exact FEM-model in the case of dynamic and high nonlinearity of structures (Figure 4.55.b).

Figure 4.55.a. Using a surrogate model in proposed optimization tools for structural optimization when objective responses show nonlinear behaviour.

Figure 4.55.b. Using an exact model for structural optimization in the case of linear response of objectives and less time-consuming FEM-analysis.
4.4.3 Evaluation of solutions using FEM

This section describes some essential settings and assumptions for analysis of the B-pillar structure in ABAQUS [82], according to the defined objectives of Table 4.4 in chapter 4.4.1. Since there are two types of load cases in this study a detailed description of the analyses will be introduced in two separate sections. Firstly for a quasi-static test of roof crush test based on FMVSS No. 216a [61] and secondly for a dynamic side crash test based on UN R95 regulation [61].

The evaluation of the remaining objectives in Table 4.4, like weight, cost, and manufacturing constraints do not need to go through FEM-analyses and are obtained from 3D model information.

Roof crush resistance

According to FMVSS No. 216 and its problem definition of "top intrusion", spring elements have been replaced [85] instead of neighbor parts of the B-pillar. The specification of these elements to achieve 70% of allowable deformation are illustrated in Figure 4.56.

![Image](image-url)

**Figure 4.56.** Roof cross-member was replaced by weaker spring of 500 N.mm and side component was replaced by stronger spring of 900 N.mm. The B-pillar creates vertical resistance. The middle section of the structure tolerates most of the applied force. The magnitude displacement at the top point is depicted in the graph.

Three-dimensional force is applied to a coupling point, which is located between inner and outer part at the top and moves 118.8 mm, in the baseline B-pillar (not optimized). The boundary condition is fixed at the bottom zone of the B-pillar and will be flexible at the top.
After applying the mentioned loads on the structure for all five groups (which were introduced in chapter 4.4.2), the amount of deformation in X, Y, and Z-direction can be obtained and recorded as one of the structural specifications, see Table 4.4.

As a rough interoperation it can be stated that structures with a stronger construction in the top middle section of B-pillar using thicker or stronger sheets or filler parts show less deformation under a roof test load.

![Images of three different structures under roof crush load](image)

**Figure 4.57.** Deflection of three different structures under roof crush load

(a) Baseline structure consisting of inner steel 2.5 mm and outer steel 3 mm
(b) Three-piece structure consist of inner steel 2 mm, lower outer steel 1.8 mm, and upper outer high strength steel 1.6 mm
(c) Al-foam filled structure consist of inner steel 2 mm, outer steel 1.8 mm, and filled by Al-foam in upper section
Side crash test

Based on the UN R95 regulation, the B-pillar has to tolerate a 950 Kg mass with 50 km/h in such a way that the destroyed parts provide a survival distance of more than 125 mm from the driver seat [61]. In this study, “point mass/inertia” has been used instead of body parts and their mass (Figure 4.38). For sure, this alternative model may not exactly represent the real behavior of B-pillar with other body parts, but it is acceptable as a comparative study to evaluate the performance of different structures. Two types of results are obtained from side crash simulation here: Intrusion of top section of B-pillar toward occupant compartment and absorbed energy by whole structure (Figure 4.58).

![Graph](image)

**Figure 4.58.** Deviation of absorbed energy during the side crash and determination of intrusion of top section towards driver seat

The amount of the mentioned results such as the stiffness of the structure against the side crash pulse are recorded in a result table (Table 4.4) for all five groups of structures. As the strain rate dependent on material properties, is not simply available, plastic deformation until damage initiation was considered for suggested materials in this study. If these data are provided, optimization results would be more reliable, but of course it also would need stronger computers and more patience.
Notations on the modelling of joints in this study

As mentioned in chapter 4.4.2 (Realization of concepts), no joining properties were assigned in this study. That means that all connections between all similar or dissimilar materials are non-damageable. However, in real situations weak connections threat the strength of structures. In order to show the importance of connection properties for structural stability, some simulation results are shown.

As an example: a roof crash load excites the baseline structure firstly with a hard connection of inner and outer part and secondly with a damageable welding connection. The structural performance of the two different connections is demonstrated in Figure 4.59.

**Figure 4.59.** Because of separation between inner and outer parts, the right structure has more deflection than the left one. Strong properties for connections will increase the structural performance and could be optimized through optimization loops in terms of connection specifications or process parameters.

As a future work of this study, connection properties like stiffness and strength in normal and shear direction for different connection technologies could be defined as input variables beside geometrical and material variables.
4.4.4 Surrogate Model for nonlinear behavior

Finite element analysis of multi-part structures like a B-pillar subjected to dynamic impactors, is a very time consuming analysis. Additionally, the structure shows relatively nonlinear behaviors for variation of input variables. Because of the large number of variables and their domains, the optimization problem has a huge design space here. Therefore, a big initial population and relatively many iterations need to find the optimum combination of variables [165, 166, 168, and 170]. On the other hand, five concepts for structural optimization in this study are considered as a small number of assumptions. In real situations, the number of concepts may increase to a two-digit number of concepts. It would need a massive increase of required simulations per optimization loop. Therefore, finding the optimum structure may take a couple of weeks and does not have any logical feasibility for designers.

In multi-objective optimization problems, there is another complexity, which increases the number of FEM-analyses as well. That is the necessity of altering the weighted ratios of objectives in order to obtain other kind of optimum results. For instance, designers may earlier want to find the stronger structure or the cheaper one. If so, he has to increase the weight ratio for strengths or cost respectively and wait for new results again. He will lose the FEM-results of every weight ratio and will not be able to use them for another amount of ratios. To overcome the mentioned problems, it seems quite logic that an approach should be able to keep the history of analyses and be able to reduce the optimization time for every kind of necessity of the designer. In this study, a surrogate model [30, 35, 77, 81, 287, and 288], has been proposed to tackle the time-consuming FEM-analyses and nonlinear behaviors.

The third order of the Least Square Regression Method (LSRM) was employed to find the model of objectives based on the variation of inputs. This approach minimizes the sum of square of errors between model and real values (as explained in chapter 2.2.1). A surrogate model was individually generated for each group of structure in order to approximate the distance of the B-pillar to the driver seat and the amount of energy absorption. The accuracy of each model was evaluated using normalized root mean square error ($R^2$), Equation 4.6 [267, 253].

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y}_i)^2} \quad (4.6)$$

$y_i$ : Calculated value of FEM for $i^{th}$ sample

$\hat{y}_i$ : Predicted value of surrogate model for $i^{th}$ sample

$\bar{y}_i$ : Mean value of FEM − results
In this study, the surrogate model is started to achieve a minimum of 90% accuracy by evaluating 50 members. Random members are selected by Latin Hypercube Sampling (LHS was explained in chapter 3.2.2). Then by calculating $R^2$, the deviation of the model from the FEM result is measured. If $R^2$ is less than 0.9, then 10 more samples are selected and evaluated. The model evaluates 60 members and the accuracy is checked again. This procedure is continued until an acceptable accuracy ($R^2 > 0.9$) is reached. After that point, the surrogate model would be reliable and can be used instead of simulation to determine the results of side crash analysis (Figure 4.60).

![Figure 4.60](image)

**Figure 4.60.** Strain energy obtained by surrogate model vs FEM-analysis for 314 samples

In order to obtain a reliable surrogate model with $R^2 > 0.9$ in this study, 300-400 analyses were executed for each group.
4.4.5 Implementation of manufacturing constraints

Manufacturing restrictions are one of the most significant reasons to mislead the structural optimization scenarios. An excellent structure, which fulfills all desired requirements of the designer like low weight, low price, and high strength, could be useless when the manufacturing restrictions have not been considered. Manufacturing limitations of parts depend on their manufacturing process [69, 205, 263, 291, and 292]. As a simple example, “no angles less than 3 degrees in plastic injection parts” is considered as one of the most important limitations. In casting and die-casting, parts avoiding thin walls are a big challenge for designers. Laminate parts have some restrictions related to the maximum continuous number of the same orientation layers along the thickness and asymmetric layer sequence [289, 293]. Ply drop laminates have some restriction in length, number, and thickness of overlapping regions, which leads altering of the design in composite structures [290].

As the B-pillar structure is mostly made of sheet parts, and this study is limited to sheet-metal parts, their formability was considered. Manufacturing limitations of composite parts have been ignored here for simplification. However, the possibility of their manufacturability could already be implemented with the proposed approach in this section for sheet metal parts.

Other important aspects, which should be considered in manufacturing limitations, are the circumstances of executing the manufacturing limitation in structural optimization loop. In other words: if a limitation is detected, how can its negative effect be considered? Before describing how a formability restriction is found and dealt with, it is useful to consider that with the current speed of new technology developments in production, it is hard to make predictions about limitations of manufacturing. The development of special materials and alloys, improvement of casting technologies, enhancing the accuracy and speed of machining, and the progress of special software to predict the manufacturing process lead to eliminate any kind of manufacturing constraints. The single challenge is to reduce the manufacturing cost not how to manufacture [189]. However, in this study one usual limitation in the sheet metal process will be discussed as a case study.

One of the most recent developments in sheet technologies is producing light and high-strength sheets. This is the exact expectation of an optimizer from a raw material in structural optimization (stiffness/weight). Composite laminate which is listed in Table 4.6 (chapter 4.4.2) and that was used in this study to construct the B-pillar, is an example for these materials. But these materials have been used with some carefulness in new structures because of the following reasons. They are currently six times more expensive than conventional materials like steels [194] and they have to connect to other materials with cohesive bounds, which needs to be incorporated in production lines beside the welding processes.
Another successful example are thin and high strength steel sheets. These materials show relatively higher yield and ultimate stresses compared to the conventional steels, but less elongation (formability). They normally need to be pre-heated before doing any mechanical works in dies and small deformations and fillets should be neglected. These limitations have been considered for two different types of high strength sheets in this study.

**Table 4.9.** Maximum allowable bending angle for Ductibor® and Usibor® families, according to VDA 238-100 [269].

<table>
<thead>
<tr>
<th>Material</th>
<th>YS (MPa)</th>
<th>UTS (MPa)</th>
<th>A (%)</th>
<th>Bending angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ductibor® 450</td>
<td>≥ 380</td>
<td>≥ 460</td>
<td>≥ 16</td>
<td>≥ 120°</td>
</tr>
<tr>
<td>Ductibor® 500</td>
<td>≥ 400</td>
<td>≥ 550</td>
<td>≥ 16</td>
<td>≥ 120°</td>
</tr>
<tr>
<td>Ductibor® 1000</td>
<td>≥ 800</td>
<td>≥ 1000</td>
<td>≥ 8</td>
<td>≥ 80°</td>
</tr>
<tr>
<td>Usibor® 1500</td>
<td>1100</td>
<td>1500</td>
<td>≥ 3</td>
<td>≥ 50°</td>
</tr>
<tr>
<td>Usibor® 2000</td>
<td>≥ 1400</td>
<td>≥ 1800</td>
<td>≥ 3</td>
<td>≥ 45°</td>
</tr>
</tbody>
</table>

*Hot rolled* | *Cold rolled*

---

5 to 10 minutes 880°C to 930°C type heat treatment followed by quenching in perfectly cooled stamping tools (cooling speed > 30°C per second).
*Point baking simulation: 170°C heat treatment during 20 minutes.*
Elongations to rupture are only indicative. More relevant is the minimum bending angle for crash ductility.
Bending angle following to the VDA238-100, referring to 1.5 mm thickness test specimen.

**Table 4.10.** Maximum allowable bending angle for the tribond® family, according to VDA 239-100 [269].

<table>
<thead>
<tr>
<th>Steel grade</th>
<th>Yield strength Rm [MPa]</th>
<th>Tensile strength Rm [MPa]</th>
<th>Elongation An [%]</th>
<th>Bending angle**</th>
</tr>
</thead>
<tbody>
<tr>
<td>tribond® 1200</td>
<td>&gt; 600</td>
<td>&gt; 1,100</td>
<td>&gt; 5</td>
<td>&gt; 130</td>
</tr>
<tr>
<td>tribond® 1400</td>
<td>&gt; 600</td>
<td>&gt; 1,300</td>
<td>&gt; 5</td>
<td>&gt; 75</td>
</tr>
</tbody>
</table>

* Mechanical properties after heat treatment depend strongly on the process parameters, the values given here apply for austenitization at 920°C and cooling > 27 K/s.
** Bending angle in acc. with VDA 238-100 plate bending test.

Cold-rolled strip
Rm: 0.2% proof stress
Rm: Tensile strength
An: Elongation of a sample with a gauge length Lg = 80 mm in thicknesses < 3.0 mm

If in the outer part (for all five groups) the wall angle exceeds the allowable bending angle of Table 4.9 and 4.10, the structure is faced with hard manufacturability. In this study, the optimization tool does not prevent that structures with non-manufacturable angles are generated, but it will take a negative score to reduce the fitness value. Figure 4.61 shows an
example of a cross-section which has a non-manufacturable angle for tribond® 1400 material from Table 4.9.

Figure 4.61. Bending angle of 85° for outer part could not be produced by normal stamping treatments and needs special pre-treatments based on the producer’s data sheet.

The number of parts that are made with non-manufacturable angles from Ductibor® and tribond® sheets are multiplied with their angle values and are recorded as a negative score in the result table. These values will be normalized and taken into account as penalty to calculate the fitness value. The wall angle is bigger than the allowable bending angle, structure takes more negative score Figure 4.61.

As previously explained, the structures with unallowable bending angles are not eliminated from the optimization process. Other properties like energy absorptions or the price being considerably better than others, can already be selected as the optimum structure. With the current speed of innovations for producing high strength steels it may not be so far from now that in the future every bending angle is achievable for high strength steels. In this optimization tools, the effect of manufacturing limits on the overall fitness of the structure can be decreased or increased.
4.4.6 Objective function and constraints handling

In chapter 2.3.2 different kinds of approaches to deal with multi-objective optimization problems have been introduced and their advantage and disadvantage were listed. As there are more than two objectives in the optimization problem of this study, both the weighted sum method and the epsilon constraint method could be employed to generate the fitness function. For instance for weighted sum method as follow [1, 2, 3, and 5].

\[ F(X) = \sum_{k=1}^{k} W_k F_k(X) \]  

(4.7)

The weighted sum method gives the possibility of obtain different optimum structures related to different weight of objectives. As an example, by increasing the weight ratio of cost in the fitness function, cheaper structures will be obtained at the end. To escape from non-convex regions, different weighted ratios near to every set of selected ratios should be assigned again. For instance, if a ratio of 0.5 is assigned to weight and cost of the structure in a multi-objective problem, it should be checked with 0.45-0.55 and 0.55-0.45 as well. As the objectives and constraints have different dimensions, it is necessary to normalize objectives between unique limits. For this reason, the minimum and maximum amount of every objective has to be determined to be used in the following normalization equation [295].

\[ F_n = \frac{(F - F_{\min})}{(F_{\max} - F_{\min})} \]  

(4.8)

Minimum and maximum amounts of objectives are obtained by sorting every column of the result table that has been fulfilled earlier by FEM-results. Here, 10% have been added to the maximum value and have been extracted from the minimum value. These minimum and maximum values were obtained from FEM-results of 300-400 structures from the surrogate model, which have been randomly selected from the whole design space.

**Table 4.11.** Real values of objectives, penalties, and fitness values of five randomly selected structures from the concept pool, generated by the optimization tool

<table>
<thead>
<tr>
<th>Group No.</th>
<th>Weight</th>
<th>Material Cost</th>
<th>Production Cost</th>
<th>Top-Intursion</th>
<th>Energy absorption</th>
<th>Manufacturing restriction</th>
<th>Penalty</th>
<th>Fitness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.5</td>
<td>31.8</td>
<td>6.0</td>
<td>207.6</td>
<td>42.1</td>
<td>458.5</td>
<td>80.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.481</td>
<td>0.616</td>
<td>0.043</td>
<td>0.559</td>
<td>0.678</td>
<td>0.132</td>
<td>0.224</td>
<td>1.656</td>
</tr>
<tr>
<td>2</td>
<td>10.1</td>
<td>23.4</td>
<td>9.0</td>
<td>240.1</td>
<td>43.1</td>
<td>655.0</td>
<td>160.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.343</td>
<td>0.417</td>
<td>0.478</td>
<td>0.665</td>
<td>0.683</td>
<td>0.206</td>
<td>0.448</td>
<td>1.425</td>
</tr>
<tr>
<td>3</td>
<td>7.9</td>
<td>18.0</td>
<td>12.0</td>
<td>221.3</td>
<td>26.8</td>
<td>213.9</td>
<td>80.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.255</td>
<td>0.288</td>
<td>0.913</td>
<td>0.604</td>
<td>0.595</td>
<td>0.039</td>
<td>0.224</td>
<td>0.604</td>
</tr>
<tr>
<td>4</td>
<td>12.6</td>
<td>18.3</td>
<td>9.0</td>
<td>193.7</td>
<td>84.7</td>
<td>165.2</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.443</td>
<td>0.296</td>
<td>0.478</td>
<td>0.515</td>
<td>0.907</td>
<td>0.021</td>
<td>0.000</td>
<td>0.957</td>
</tr>
<tr>
<td>5</td>
<td>10.4</td>
<td>25.5</td>
<td>9.0</td>
<td>152.5</td>
<td>40.9</td>
<td>804.4</td>
<td>165.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.355</td>
<td>0.467</td>
<td>0.478</td>
<td>0.381</td>
<td>0.671</td>
<td>0.262</td>
<td>0.462</td>
<td>1.202</td>
</tr>
</tbody>
</table>
In the last column, Table 4.11 shows, the fitness value of every structure using the same weighted ratios for all objectives.

From the demonstrated structures in Table 4.11, the three-piece structure has a minimum fitness value and consequently the best one. It should be noted that the fitness function of this study is a minimization type. Therefore, some objectives, which must be maximized during the optimization loop (absorbed energy and distance of B-pillar to driver seat), should be converted to minimum, using one of min/max conversation approaches. In this study a normalized amount of mentioned objectives was extracted to reverse the effect of their increase in fitness function.

**Constraints handling (penalties)**

To implement the negative effect of objectives, which are out of range in Table 4.1 a value proportion to the amount of deviation, will be added to the fitness function. Increasing the fitness value means that the structure has less chance to participate in optimization loops [65, 217]. Equation 3 shows the fitness function with elastic penalty scheme.

\[
F(X) = F(X) + r \sum_{j=1}^{m} \{\max[0, g_j(X)]\}^2
\]  

(4.9)

A high amount of \(r\) increases the violation of penalties and causes an earlier elimination of structure [2]. Objectives in this study are added to the penalty, if following conditions are satisfied:

- If weight of structure is higher than 10.08 kg (80% of baseline weight).
- If cost of structure is more expensive than 30.24 Euro (20% of baseline cost).
- If top-intrusion is greater than 127 mm (FMVSS No. 216a).
- If intrusion of the B-pillar to the driver seat is less than 125 mm (UN R95).

The effect of manufacturing limitations have been implemented as penalty function as well (Table 4.4, column 8). If one structure has a non-manufacturable bending angle (infeasible), an amount which depends on its bending angle, will be added to the fitness function to reduce the chance of the structure contesting with others.
4.4.7 Comparative study on optimization results

This section demonstrates the optimization result of the B-pillar structure with different architectures of objective function. A net graph was used to introduce the priority of objectives to each other. Since in the optimization of mechanical structures (like this study) usually more than two objectives are simultaneously involved, it is hard to determine a specific structure as the global optimum. As it will be demonstrated on the next pages, a small change in objective priority leads to obtain a different group of structures. However, in this study, the surrogate model simplifies this complexity. With the proposed optimization tool, the user is able to simply change the fitness function strategy into weighted sum method, epsilon-constraint method, and Pareto optimum approach as well as altering the constraint violations using elastic constrain approach. The user also has the opportunity to assign different weight ratios (priorities) to every objective and obtain related optimum results in a relatively short time. If the user is not sure about the priority of objectives and all specifications of the structure have the same importance, the weighted sum method is suggested. When one objective is clearly more important than the other, the epsilon constraint method helps to consider less relevant specifications as constraint and gradually deletes infeasible structure through the iterations. In the case of two objectives with the same importance, the user could select the Pareto front approach and obtain an optimum structure, which fulfils both requirements.
The best structure, which fulfils all objectives simultaneously

Considering the problem as an unconstrained problem and using weighted sum method with the same ratio for all objectives, the tool offers an Al-foam filled structure as the optimum structure.

**Figure 4.62.** All specifications have been considered as objectives with the same priority

<table>
<thead>
<tr>
<th>Specification</th>
<th>Baseline value</th>
<th>Objective/Constraint</th>
<th>Optimum*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kg)</td>
<td>20.31</td>
<td>&lt; 80% Baseline</td>
<td>12.9</td>
</tr>
<tr>
<td>Total Cost (€)</td>
<td>24.37</td>
<td>&lt; 150% Baseline</td>
<td>31.90</td>
</tr>
<tr>
<td>Top-Intrusion (mm)</td>
<td>118.8</td>
<td>&lt; 127 mm</td>
<td>62.7</td>
</tr>
<tr>
<td>Intrusion B-pillar to driver seat (mm)</td>
<td>130</td>
<td>&gt;125 mm</td>
<td>171</td>
</tr>
<tr>
<td>Energy absorption (N.m)</td>
<td>450</td>
<td>minimize</td>
<td>1281</td>
</tr>
<tr>
<td>Manufacturing restrictions</td>
<td>no value</td>
<td>minimize</td>
<td>0</td>
</tr>
</tbody>
</table>

* same priority for all objectives
**Altering of dimensions, materials, and thickness during the iterations**

Finding the best combination of variables for all concepts was carried out in parallel, using the proposed optimization tool. At the end, concept 4 (Al-foam filled) represented the minimum fitness and was selected as the optimum structure. For the optimum structure, it took thirty-eight iterations to converge into minimum fitness value. Figure 4.63 shows how the optimization tool changed the material of the inner, outer, and filler part and related their thickness to obtain the global optimum.

![Figure 4.63](image)

**Figure 4.63.** Baseline structure is optimized with solution number 4, i.e. using filler material. Length and position of Al-foam has more influence than cross-sections and thickness of parts to increase the structural properties in this solution. (same priority for all objectives has been assumed in this case).
Comparative study on using proposed adaptive hybrid approach

As shown in section 3.3, Table 3.3, proposed adaptive optimization algorithm required less cost to find the global optimum compare to the non-adaptive algorithm with a given number of reliability. Whereas an unreliable algorithm has higher probability to find a local optimum instead of a global one. Reliability was obtained by dividing the number of runs, which have found any given value of global optimum per total accomplished runs [187, 188]. Here in optimization of B-pillar, the effectiveness of adaptive search approach to reduce the cost of optimization algorithm is evaluated. Amount of reliability has been set on 90%.

**Table 4.12.** Comparison of costs for adaptive and non-adaptive optimization algorithm in B-pillar optimization problem.

<table>
<thead>
<tr>
<th>Type of optimization algorithm</th>
<th>Cost*</th>
</tr>
</thead>
<tbody>
<tr>
<td>non-adaptive</td>
<td>1.00</td>
</tr>
<tr>
<td>adaptive</td>
<td>0.86</td>
</tr>
</tbody>
</table>

*Cost of algorithms has been normalized by greater cost value

From Table 4.12 it can be obtained that using of adaptive algorithm has reduces the optimization cost up to 14%. To get an overall idea about total CPU time, required number of simulations are separately reported for five predefined concepts in Table 4.13.

**Table 4.13.** Number of simulations to find the optimum structure for every group of concepts (totally 2860 simulations). Simulation of side crash has the majority of simulations number. Every group needed 30-40 iterations to converge (Figure 4.63).

<table>
<thead>
<tr>
<th>Description of concept</th>
<th>Number of simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-part assembly</td>
<td>460</td>
</tr>
<tr>
<td>Three-part assembly</td>
<td>520</td>
</tr>
<tr>
<td>Four-part assembly</td>
<td>540</td>
</tr>
<tr>
<td>Foam reinforced assembly</td>
<td>680</td>
</tr>
<tr>
<td>Ribs reinforced assembly</td>
<td>660</td>
</tr>
</tbody>
</table>
Optimum structure with the priority on weight reduction

Considering the problem as a constrained problem and using the epsilon-constraint method with the objective of weight reduction, the tool offers a three-piece structure as the best.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Baseline value</th>
<th>Objective/Constraint</th>
<th>Optimum*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kg)</td>
<td>20.31</td>
<td>&lt; 80% Baseline</td>
<td>10.2</td>
</tr>
<tr>
<td>Total cost (€)</td>
<td>24.37</td>
<td>&lt; 150% Baseline</td>
<td>29.84</td>
</tr>
<tr>
<td>Top-Intrusion (mm)</td>
<td>118.8</td>
<td>&lt; 127 mm</td>
<td>111.6</td>
</tr>
<tr>
<td>Intrusion B-pillar to driver seat (mm)</td>
<td>130</td>
<td>&gt;125 mm</td>
<td>138</td>
</tr>
<tr>
<td>Energy absorption (N.m)</td>
<td>450</td>
<td>minimize</td>
<td>858</td>
</tr>
<tr>
<td>Manufacturing restrictions</td>
<td>no value</td>
<td>minimize</td>
<td>0</td>
</tr>
</tbody>
</table>

* more priority on lighter structure and less priority on other specifications

**Figure 4.64.** Only weight reduction has been considered as objective and other specifications have been considered as constraint
Optimum structure with priority on weight reduction and low price

Considering the problem as a multi-objective constrained problem and assuming both weight and cost as main objectives, the tool offers a three-piece structure as the best Figures 4.65.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Baseline value</th>
<th>Objective/Constraint</th>
<th>Optimum*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (kg)</td>
<td>20.31</td>
<td>&lt; 80% Baseline</td>
<td>11.8</td>
</tr>
<tr>
<td>Total Cost (€)</td>
<td>24.37</td>
<td>&lt; 150% Baseline</td>
<td>26.72</td>
</tr>
<tr>
<td>Top-Intrusion (mm)</td>
<td>118.8</td>
<td>&lt; 127 mm</td>
<td>113.1</td>
</tr>
<tr>
<td>Intrusion B-pillar to driver seat (mm)</td>
<td>130</td>
<td>&gt;125 mm</td>
<td>135.8</td>
</tr>
<tr>
<td>Energy absorption (N.m)</td>
<td>450</td>
<td>minimize</td>
<td>841</td>
</tr>
<tr>
<td>Manufacturing restrictions</td>
<td>no value</td>
<td>minimize</td>
<td>0</td>
</tr>
</tbody>
</table>

* more priority on lighter and cheaper structure

**Figure 4.65.** Reduction of weight and cost have been considered as multi-objective and other specification have been considered as constraint
Improvement of optimum result

In order to investigate the improvement of the optimized structure, normalized values of objectives for baseline and optimum structure are introduced in Figure 4.66.

![Figure 4.66](image_url)

**Figure 4.66.** Improvement of specifications after optimization with the proposed structural optimization tool of this study.
The next optimum structure from other groups of concepts

Al-foam filled structures have proved to be the optimum structures to fulfil all objectives. However, investigating optimum structures from other groups of initial concepts might be useful. In the current study, after Al-filled structures, Ribs-filled structures have the optimum fitness values.

The results of column 4 and 5, only show small differences in their specifications and it is hard to select one of them as the optimum structure. Some additional criteria need to make the final decision in these cases. For instance, a company without aluminium foam facility may prefer to consider the Ribs-filled structure at the end.

**Figure 4.67.** Comparison of the objective values of optimum Al-foam and Ribs-filled structure in the last columns.
As the energy absorption is considered as one of the significant objectives in the P-pillar optimization here, the improvement of absorbed energies by Al-foam and Ribs-Filled structures are compared with the baseline value.

**Figure 4.68.** Optimum structures absorb the side crash energy almost three times better than the baseline structure
Both Al-foam and Ribs fillers reduce the excessive deformation of the outer part and the structure absorbs the impact energy with a smaller deformation. But in this study, because of using the relatively wide contact surface of the impactor (based on UN R95), this deformation is not clearly visible. Therefore, here a sharper impactor with less contact surface has been selected for demonstration (Figure 4.69).

![Figure 4.69](image1.png)

**Figure 4.69.** Right picture shows the deformation of non-filled structure and left picture shows smaller deformation of filled structure with Al-Foam

This type of impactor was not used in this study and it was demonstrated only to show the better performance of filled structures than the non-filled ones. ThyssenKrupp has used a similar type of impactor to evaluate tribond® 1400 material to improve the B-pillar performance [279] (Figure 4.70). Filler materials could be made of metal or CFRP stiffeners and are attached to the outer parts by welding or cohesive materials.

![Figure 4.70](image2.png)

**Figure 4.70.** Evaluation of high strength steel using a special impactor form [279]
5. Conclusion and outlook

5.1 Conclusion

This study introduced a methodology to decrease the weight and increase the mechanical properties of sheet-made structures under several load cases, manufacturing limitations and financial restrictions. A FEM-based optimization tool finds the optimum combination of geometrical and material combinations to achieve the predefined objectives. The necessity for an efficient and strong optimization tool is given when a sheet metal structure is supposed to become lighter without decreasing its strength and with considering project limitations. Therefore, lightweight design was selected as an applicable solution with some complexities and challenges to implementation. These complexities were divided into smaller groups to be easier to solve.

The first challenge occurred when using light materials increased the structure cost up to 6 times. A well-known solution is to use light/strength materials only in crucial places of the structure. This solution caused the problem of finding the best combination of materials throughout the structure. It was shown that finding the right material combination without considering the variation of cross-sections and thickness is meaningless. Since all structures of car bodies, which are selected for optimization, should be modelled in a 3D-model in CAD, the optimization approach of this study was designed based on the parametrization of the initial concept in CAD. Therefore, not only all possible concepts consist of geometrical variations and material combinations were considered, but also the necessity for surface smoothing of optimum structure was eliminated.

The second difficulty in structural optimization was introduced in terms of multi-load case problems. Finding the optimum combination of variables without considering all subjected loads leads towards unreliable results at the end. In order to overcome this problem, a methodology was proposed between CAD and CAE to generate the input file consisting of meshed structure, boundary conditions, load cases, and joints as defined earlier by the user. As evaluation of structures under dynamical load cases is considered as unpredictable behavior and a very time-consuming process, a surrogate model was proposed instead of exact FEM-analyses during the iterations in optimization tool. In this way, predefined concepts for structural improvements were realized and evaluated and the related results were stored automatically.

The third complexity, i.e. the inseparable scenarios in structural design, are manufacturing and financial limitations. Different types of limitations were introduced and two types of them, i.e. manufacturing limitations and cost restrictions, were selected for investigation. Formability i.e. allowable bending angles for high strength steels, as one of the major limitations in sheet metal...
manufacturing, were determined and implemented in the optimization process. Structures who exceed the recommended bending angles of the manufacturer, have less chances to participate in the optimization loop by adding additional terms to their fitness function. To consider the extra production cost of multi-parts structures, a cost model was proposed to add the cost of machines, molds, and joints to the material cost. Thus, multi-piece structures may come off worse even if they show better mechanical performance than structures with less pieces.

The fourth problem was the existence of many local optima in structural optimization due to the discretization of input variables. It was shown that optimization algorithms are already at risk to be stuck in a local optimum instead of a global one. An adaptive hybrid metaheuristic was proposed, which make use of the advantages of some explorative and exploitative features of heuristics to reduce the risk of finding semi-global optima. Thereby, all possible ways towards a local optimum could be detected and appropriate reaction/s were carried out to adjust the optimization direction.

The optimization tool, which was designed to overcome the mentioned complexities, was used to optimize the B-pillar performance under quasi-static and crash load cases. Five multi-material concepts were realized in CAD and optimum solutions were found using FEM-analyses and hybrid optimization algorithm. At the end, the global optimum and some local optima with different architectures were compared using different multi-objective approaches.
5.2 Outlook

As much as this study tried to precisely breakdown the structural optimization problem and determine all complexities and their related solutions, but some regions still need to be developed to enhance the performance of the proposed optimization tool.

- In this work, five concepts based on hybridization of traditional structures were developed to increase the structural performance. It could be possible to evaluate more concepts beside the current approaches, such as using tailor rolled metal sheets, local fiber patches, and stiffeners.
- It is possible to combine different concepts, which was ignored in this study for simplification. Combinations of two or more concepts may introduce a better performance than the individual ones.
- The connection of similar or dissimilar parts was considered as non-damageable joints in this study. It would be reasonable to define cohesive, welded or bolted joints with different mechanical properties between parts in future. By this way, not only FEM-results become more reliable, but also optimum connection properties will be obtained beside the geometry and material combinations during the optimization loops.
- The optimized structure in this study was subject to two load cases. It could be possible that more load cases are added to optimization process like fatigue loads [70], temperature effects, force of seat belt in frontal crash events etc. without any modification in the optimization tool.
- This study considered the manufacturing restrictions due to the difficult formability of high strength steel sheets. Manufacturability of parts with composite laminate [157, 290] and plastic injection parts could be also taken into account in future works.
- In FEM-analyses ABAQUS software was employed. With some small modifications other finite element software like PAM-Crash [116] and LS-DYNA [148] can be employed as they may show better flexibility in crash simulations.
- A specific stacking sequence for composite sheets based on manufacturer recommendations was considered in this study. However, as a precise design strategy, it would be possible to simultaneously optimize the number, orientation, thickness, and material of layers in laminate parts.
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## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFP</td>
<td>Automated Fiber Placement</td>
</tr>
<tr>
<td>AHMA</td>
<td>Adaptive Hybrid Metaheuristic Algorithm</td>
</tr>
<tr>
<td>AIA</td>
<td>Artificial Immune Algorithm</td>
</tr>
<tr>
<td>ANOVA</td>
<td>Analysis Of Variance</td>
</tr>
<tr>
<td>BFO</td>
<td>Bacterial Foraging Optimization</td>
</tr>
<tr>
<td>BIW</td>
<td>Body In White</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>CAE</td>
<td>Computer Aided Engineering</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon Fiber Reinforced Polymer</td>
</tr>
<tr>
<td>$CO_2$</td>
<td>Carbon Dioxide</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>CS</td>
<td>Cuckoo Search</td>
</tr>
<tr>
<td>DE</td>
<td>Differential Evolution</td>
</tr>
<tr>
<td>DOE</td>
<td>Design Of Experience</td>
</tr>
<tr>
<td>ECU</td>
<td>Electronic Control Unit</td>
</tr>
<tr>
<td>FA</td>
<td>Firefly Algorithm</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FUSS</td>
<td>Fitness Uniform Selection Scheme</td>
</tr>
<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>LHS</td>
<td>Latin Hypercube Sampling</td>
</tr>
<tr>
<td>LSM</td>
<td>Least Square Method</td>
</tr>
<tr>
<td>MHS</td>
<td>Metaheuristics</td>
</tr>
<tr>
<td>MLSM</td>
<td>Moving Least Square Method</td>
</tr>
<tr>
<td>MOP</td>
<td>Multi-Objective Problem</td>
</tr>
<tr>
<td>NLPQL</td>
<td>Non-Linear Programming By Quadratic Lagrangian</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>NSGA</td>
<td>Non Dominated Sort Genetic Algorithm</td>
</tr>
<tr>
<td>OPRM</td>
<td>Optimal Polynomial Regression Model</td>
</tr>
<tr>
<td>PMHS</td>
<td>Population-Based Metaheuristics</td>
</tr>
<tr>
<td>POD</td>
<td>Proper Orthogonal Decomposition</td>
</tr>
<tr>
<td>PSO</td>
<td>Particle Swarm Optimization</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
</tr>
<tr>
<td>RSM</td>
<td>Response Surface Method</td>
</tr>
<tr>
<td>RTM</td>
<td>Resin Transfer Molding</td>
</tr>
<tr>
<td>SA</td>
<td>Simulated Annealing</td>
</tr>
<tr>
<td>SCM</td>
<td>Sheet Molding Compounds</td>
</tr>
<tr>
<td>SIMP</td>
<td>Solid Isotropic Microstructure With Penalization</td>
</tr>
<tr>
<td>SMHS</td>
<td>Single-Solution Metaheuristics</td>
</tr>
<tr>
<td>SQP</td>
<td>Sequential Quadratic Progressing</td>
</tr>
<tr>
<td>VUMAT</td>
<td>User Defined Material In Explicit Solver</td>
</tr>
<tr>
<td>ZDT</td>
<td>Zitzler, Deb And Thiele</td>
</tr>
</tbody>
</table>
List of publications

Throughout the elaboration of this thesis, intermediate results were published in following papers:


2 Ghaffari Mejlej Vahid, Türck Eiko, Vietor Thomas: Finding the best material combinations through multi-material joining, using genetic algorithm, ECCM17, Munich 2017


Berichte aus dem Institut


Berichte aus dem Institut 265


55 Kaletka, I. “Zielgerichtetes Entwickeln im Methodischen Konstruktionsprozeß


Lu, W. “A Model-based Approach for Robust Design in the Conceptual Phase


Die Berichte Nr. 1-36 gehen aus dem Institut für Konstruktionslehre, Maschinen- und Feinwerkelemente hervor, die Berichte 37-91 aus Institut für Konstruktionstechnik. Die Berichte sind, sofern kein Verlag angegeben ist, wie folgt zu beziehen:

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