(K_5) resp. (T_5) is equivalent to the fact that all first resp. all the other denominators are different from 0. According to (11), these 30 functions add up to 6; the reader is invited to check this directly. We have

$$F_{12}(x,y) = (1-y)\ \frac{v-y}{v}\,\frac{q-y}{q},\ F_{13}(x,y) = (1-x)\ \frac{u-x}{u}\,\frac{p-x}{p},$$

$$F_{14}(x,y) = \frac{v(1-x)+uy}{v}\,\frac{vx+u(1-y)}{u}\,\frac{v(p-x)-u(q-y)}{vp-uq},$$

$$F_{15}(x,y) = \frac{q(1-x)+py}{q}\,\frac{qx+p(1-y)}{p}\,\frac{q(u-x)-p(v-y)}{qu-pv},$$

$$F_{23}(x,y) = (x+y)\ \frac{u+v-x-y}{u+v-1}\,\frac{p+q-x-y}{p+q-1},$$

$$F_{24}(x,y) = \frac{vx+(1-u)y}{v}\,\frac{vx+(1-u)(y-1)}{u+v-1}\,\frac{v(p-x)+(1-u)(q-y)}{v(p-1)+(1-u)q},$$

$$F_{25}(x,y) = \frac{qx+(1-p)y}{q}\,\frac{qx+(1-p)(y-1)}{p+q-1}\,\frac{q(u-x)+(1-p)(v-y)}{q(u-1)+(1-p)v},$$

$$F_{34}(x,y) = \frac{(1-v)x+uy}{u}\,\frac{(1-v)(x-1)+uy}{u+v-1}\,\frac{(1-v)(p-x)+u(q-y)}{(1-v)p+u(q-1)},$$

$$F_{35}(x,y) = \frac{(1-q)x+py}{p}\,\frac{(1-q)(x-1)+py}{p+q-1}\,\frac{(1-q)(u-x)+p(v-y)}{(1-q)u+p(v-1)},$$

$$F_{45}(x,y) = \frac{(q-v)x+(u-p)y}{qu-pv}\,\frac{(q-v)(x-1)+(u-p)y}{q(u-1)+(1-p)v}\,\frac{(q-v)x+(u-p)(y-1)}{(1-v)p+u(q-1)}.$$

According to (13), these 10 functions add up to 4; the reader is invited to check this directly.

By the way, we worked the Examples $n=4$, $n=5$ first and looked for a proof of (11) afterwards.

References

[1] G.J. Rieger, On interpolation in 3-dimensional space. Portugaliae Math. **30** (1971), 119–136.