

A similar result holds for $\wp'(w)$,

$$\left\{ \Gamma(s) \sum_{\omega \in \Omega} [|\omega + 2w|^{-2s} - 4|\omega + w|^{-2s}] \right\} \Big|_{s=0} = -\log \left[\left| \frac{\wp'(w)}{\Delta(E)^{1/4}} \right|^2 \right].$$

In particular, if g_2 and g_3 are integral, and w and w_0 are N -division points with neither w nor w_0 nor $w + w_0$ nor $w - w_0$ in Ω , or simply $2w$ is not in Ω in the second case, then

$$\frac{\wp'(w) - \wp'(w_0)}{\Delta(E)^{1/6}} \text{ and } \frac{\wp'(w)}{\Delta(E)^{1/4}}$$

are $N\Delta$ -units with the indicated improvements holding for N having at least two distinct prime factors or $j(z)$ being integral. These numbers even arise from the first derivative at $s=0$ of generalized Dirichlet series as the formulas above show. Whether or not they also arise naturally as units in my conjectures is not known, but we have, at the very least, an extremely interesting supply of new units to think about.

References

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