# WATER WAVE TRANSFORMATION DUE TO DOPPLER's PRINCIPLE

by

Fritz BÜSCHING

Preprint from
Heft 65 (1979) der Mitteilungen des
LEICHTWEISS-INSTITUTS für Wasserbau der
TECHNISCHEN UNIVERSITÄT BRAUNSCHWEIG
- ISSN 0343-1223 -

Because of incorrect reasoning the actual publication of the article had been prevented previously.

CONTENTS

# INHALTSVERZEICHNIS

## 1. Introduction

In a conservative treatment of water wave transformation due to decreasing water depth the wave period T is assumed to be a constant value, whereas wave heights H, lengths L and phase velocities c change.
In addition to this it is well known that real surface waves continuously change form as they proceed into shallower water in such a way that on one hand they get increasingly asymmetric (BIESEL, 1951 ; PATRICK and WIEGEL, 1956 ; ADEYMO, 1968 ; IWAGAKI, SAKAI and KAWASHIMA, 1973 ; FÜHRBÖTER, 1974) and on the other hand the initial waves "decompose" into several smaller and shorter water level deflexions called solitons (MULTER and GALVIN, 1967 ; MADSEN and MEI, 1969 ; ZABUSKY and GALVIN, 1971 ; GALVIN 1972).

As, however, solitons can not be considered in using a wave analysis based on the zero-up-crossing or similar evaluation techniques this kind of evaluation method appeared to be rather questionable to be applied to measurements in the near shore area, in particular because the derived mean wave period turned out to be increasing with the water depth decreasing, see Fig. 1.

This phenomenon can, however, be regarded as true, since it also appeared in measured near shore wave spectra particularly in the surf zone (BÜSCHING, 1974 and 1976), see Fig. 2 and in the swash zone (SONU, PETTIGREW and FREDERICKS, 1974).

Since this effect could only be described for a long time, now the author is convinced to have regognized a mechanism of theoretical wave transformation which was not considered in the past in this respect and moreover only with reference to frequency shifts due to tidal streams (BARBER and URSELL, 1948).

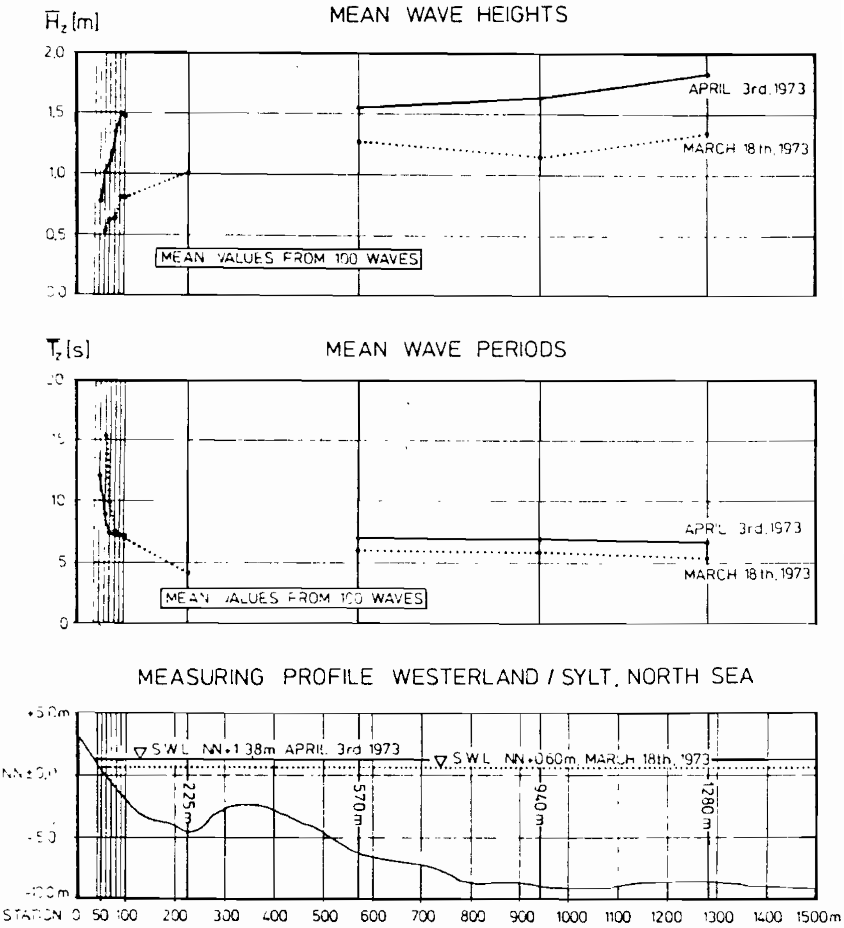


Fig. 1 : Mean wave heights and periods in the measuring profile on the isle of SYLT/North Sea as a result of strip chart evaluations based on the zero-up-crossing

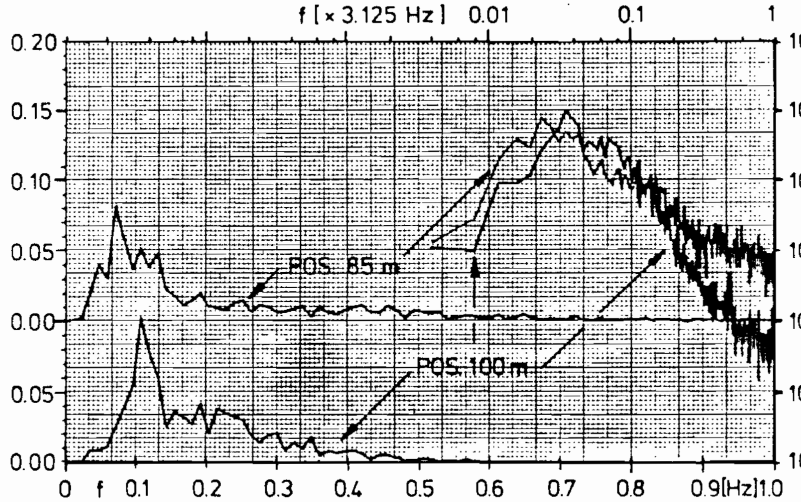


Fig. 2 : Normalized energy spectra of surf waves at distances of 85 m and 100 m from the shore of the isle of SYLT plotted in a linear and logarithmic presentation respectively

Fortunately this kind of phenomena can be explained without any speculation or unknown theory on the basis of DOPPLER's principle and the problematic nature of the following treatment of water wave transformations and derived phenomena can be regarded as a sort of analogy to the interpretation of the red shift in extra-galactic nebulae by HUBBLE (1929) with reference to the well known theory of an expanding universe.

## 2. The DOPPLER-EFFECT as a reason for water wave transformations

In accordance with the terminology of DOPPLERS's principle let a wave spectrum $G\eta_0\eta_0(f)$ or a representative wave period $T_0$ be assigned to a source of vibrations (transmitter or wave generator) at a distance of $x_0$ from the shore, whereas a wave measuring station (receiver) is located at a position denoted x, see Fig. 3.

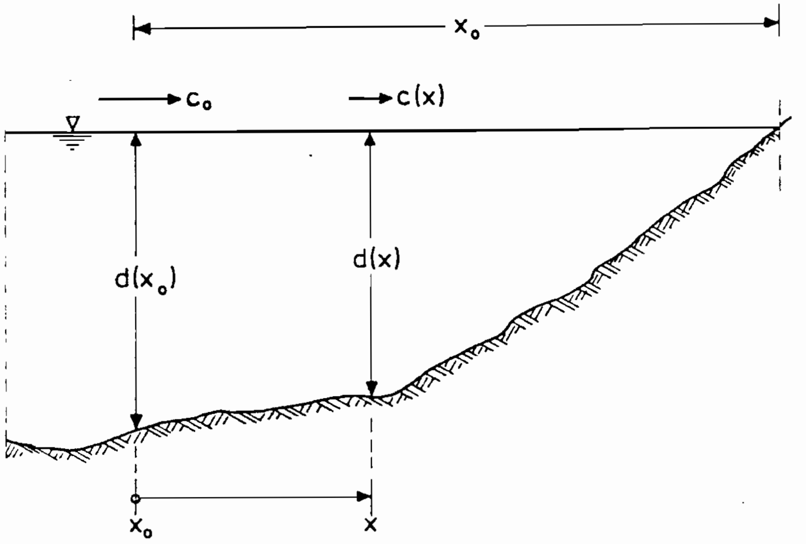


Fig. 3 : Definition sketch

Contrary to the actual definition of DOPPLER's principle the following idea is, however, not based on a transmitter moving with reference to a fixed receiver but vice versa it is proceeded on the assumption of a variable "transmitting velocity" (wave propagation velocity) whereas transmitter and receiver are stationary.

This kind of velocity changes of surface gravity waves can be due to

a) the influence of underwater bathymetry as defined theoretically by various dispersion relationships and

b) accelerated currents of any kind superimposed on the actual wave kinematics.

In general both influences will be important for transitional and shallow water conditions, whereas underwater bathymetry of course can be neglected on deep water.

If there is a velocity gradient between positions $x_o$ and x with the result of a lower phase velocity c(x) , it can be inferred that less oscillations (waves) will arrive at the receiver during a time unit as if $c(x) = c_o$ were constant.

The wave length radiated from the transmitter

$$L_0 = \frac{c_0}{f_0} = c_0 \cdot T_0 \qquad (1)$$

appears to be elongated by

$$\frac{c_0 + \Delta c}{c_0} \qquad (2)$$

i. e., the wave length measured by the receiver is

$$L(x) = \frac{L_0}{c_0} \cdot (c_0 + \Delta c) = \frac{1}{f_0} (c_0 + \Delta c) \qquad (3)$$

Furthermore it is

$$c(x) = c_0 - \Delta c ; \qquad \Delta c = c_0 - c(x) \qquad (4)$$

and as a result from (3) and (4) it is found at position x :

$$f(x) = \frac{c(x)}{L(x)} = f_0 \frac{c(x)}{c_0 + \Delta c} = f_0 \frac{c(x)}{2c_0 - c(x)} \qquad (5)$$

If the frequency shift is defined by

$$\Delta f = f(x) - f_0 \qquad (6)$$

we get

$$\Delta f = f_0 \frac{2(c(x) - c_0)}{2c_0 - c(x)} \qquad (7)$$

or in the form of a dimensionless frequency shift

$$\frac{\Delta f}{f_0} = \frac{2(c(x) - c_0)}{2c_0 - c(x)} \qquad (8)$$

Accordingly $\Delta f$ can be positive or negative depending on

$$2c_0 > c(x) > c_0 \qquad (9a)$$

or

$$2c_0 < c(x) < c_0 \qquad (9b)$$

For an accelerated movement referring to position $x_o$ the following system of equations can be written down :

$$\ddot{x} = b; \ \dot{x} = bt + c_0; \ x = \frac{bt^2}{2} + c_0 t \qquad (10)$$

If time is eliminated from

$$\dot{x} = c(x) = bt + c_0 \qquad (11)$$

it results

$$c(x) = \sqrt{c_0^2 + 2bx} \qquad (12)$$

Inserted in (8) in general we get for known accelerations b :

$$\frac{\Delta f}{f_0} = \frac{2\,(\sqrt{c_0^2 + 2bx} - c_0)}{2c_0 - \sqrt{c_0^2 + 2bx}} \qquad (13)$$

with finite real positive frequency shifts for

$$-\frac{3}{4}\,\frac{c_0^2}{2x} < b < 3\,\frac{c_0^2}{2x} \qquad (14\ a)$$

and finite real negative frequency shifts for

$$-\frac{3}{4}\,\frac{c_0^2}{2x} > b > -\frac{c_0^2}{2x} \quad \text{and} \qquad (14\ b)$$

$$\frac{3}{2}\,\frac{c_0^2}{2x} < b \qquad (14\ c)$$

For the time being in the following the influence of underwater bathymetry shall be considered only with respect to the first order dispersion relationship (AIRY), which describes a so-called normal dispersion

$$\frac{dc}{dL} \geq o \ \text{and} \ \frac{dc}{df} \leq o \qquad \text{respectively} \qquad (15)$$

This dispersion relationship is given with reference to the coordinate x as follows

$$c(x) = L(x) \cdot f(x) = \frac{L(x)}{T(x)} = \left(\frac{gL(x)}{2\pi} \tanh \left[\frac{2\pi d(x)}{L(x)}\right]\right)^{1/2} \quad (16)$$

and it is plotted in Fig. 4 in the frequency range $0 < f \leq 0.5$ Hz as a family of curves $c(f)$ marked by the parameter of the water depth d on the left hand side.

Equation (7) becomes :

$$\Delta f = f_0 \frac{2\left[\sqrt{\frac{gL(x)}{2\pi} \tanh \left[\frac{2\pi d(x)}{L(x)}\right]} - c_0\right]}{2 c_0 - \sqrt{\frac{gL(x)}{2\pi} \tanh \left[\frac{2\pi d(x)}{L(x)}\right]}} \quad (17)$$

and with reference to a constant slope (see Fig. 5) defined by

$$\gamma = 1 : n = \tan \alpha = \frac{d(x)}{x_0 - x} \quad \text{or} \quad d(x) = \frac{x_0 - x}{n} \quad (18)$$

respectively we get

$$\Delta f = f_0 \frac{2\left[\sqrt{\frac{gL(x)}{2\pi} \tanh \left[\frac{2\pi (x_0 - x)}{n \cdot L(x)}\right]} - c_0\right]}{2 c_0 - \sqrt{\frac{g \cdot L(x)}{2\pi} \tanh \left[\frac{2\pi (x_0 - x)}{n \cdot L(x)}\right]}} \quad (19)$$

As there are $c(x)$ and $f(x)$ still contained in (17) and (19) because

$$L(x) = \frac{c(x)}{f(x)} \text{ and } \Delta f = f(x) - f_0 \quad (20)$$

a consistent solution cannot be given at present.

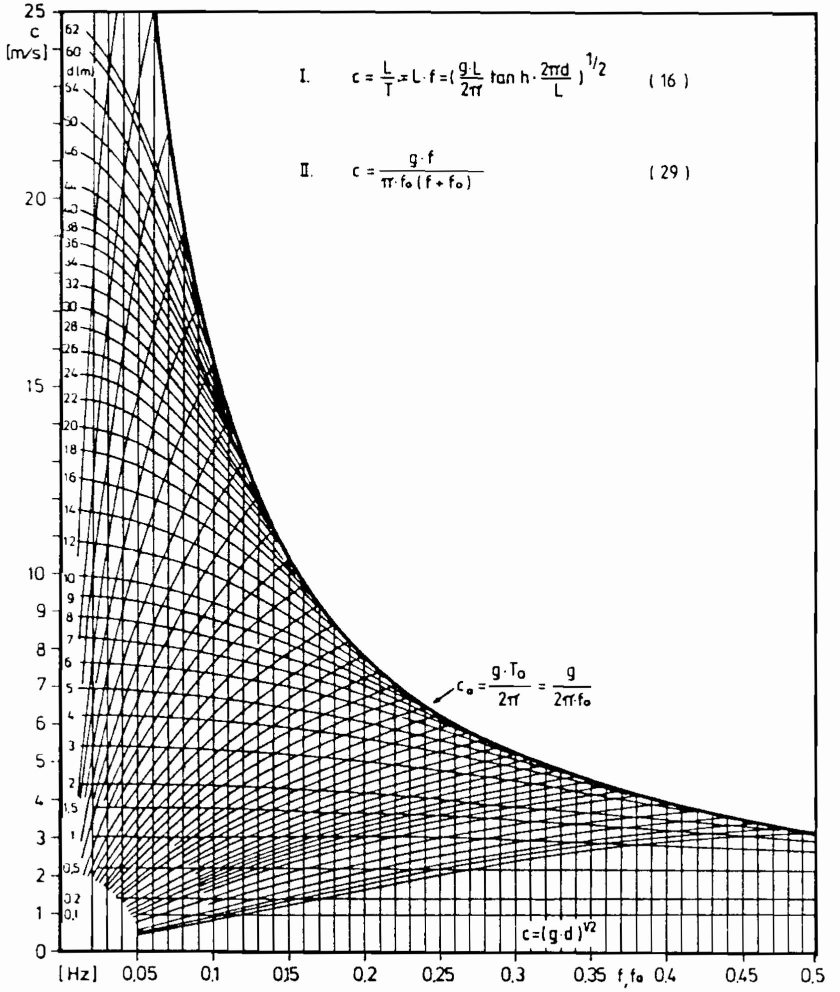


Fig. 4 : Family of curves I : (dc / df $\leq$ 0 ; parameter d)
First order dispersion relationship
Family of curves II : (dc / df $\geq$ 0 ; parameter $f_o$)
Wave beams

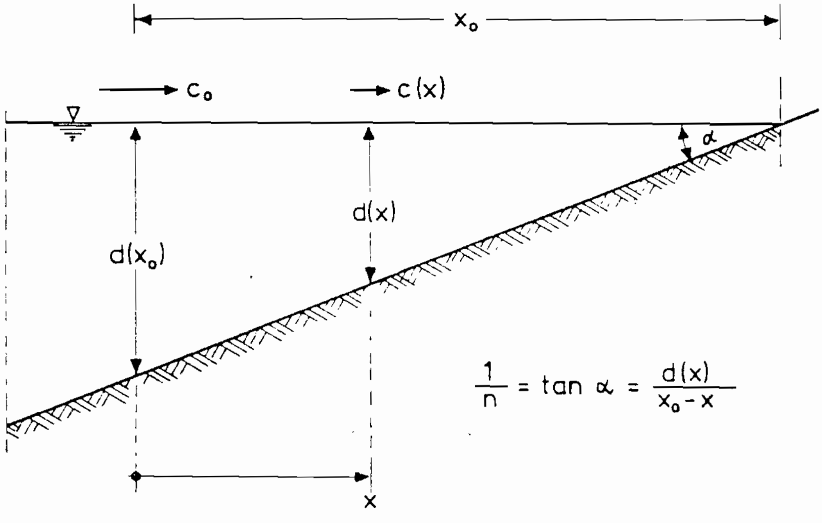


Fig. 5 : Definition sketch

This can, however, easily be obtained for small values of $d(x)/L(x)$ , provided that

$$\tanh \frac{2\pi d}{L} = \frac{2\pi d}{L} \tag{21}$$

at

$$\frac{dc}{df} = \frac{dc}{dL} = 0 \tag{22}$$

With reference to Fig. 5 the shallow water equation is

$$c(x) = \sqrt{gd(x)} = \sqrt{\frac{g}{n}(x_0 - x)} \tag{23}$$

yielding in this case frequency shifts

$$\frac{\Delta f}{f_0} = \frac{2\left(\sqrt{gd(x)} - c_0\right)}{2\, c_0 - \sqrt{gd(x)}} \tag{24}$$

and

$$\frac{\Delta f}{f_0} = \frac{2\left(\sqrt{\frac{g}{n}(x_0 - x)} - c_0\right)}{2\,c_0 - \sqrt{\frac{g}{n}(x_0 - x)}} \qquad (25)$$

respectively.

At decreasing water depth in line with positive x-direction it is obvious that always negative frequency shifts (red shifts) will be obtained for

$$x < x_0$$

and $$c_0 = \sqrt{g d_0} = \sqrt{\frac{g}{n}\,x_0} \qquad (26\ a)$$

whereas positive frequency shifts (blue shifts) will result at increasing water depth for

$$-\,3x_0 < x < 0 \qquad (26\ b)$$

Furthermore red shifts could be expected, too at

$$-\,3x_0 > x \qquad (26\ c)$$

if the shallow water equation were applicable at this x-range, however, this is not the case.

If in equation (17) and (18) respectively reference is made to deep water conditions characterized by

$$c_0 = \frac{g\,T_0}{2\pi} = \frac{g}{2\pi f_0} \qquad (27)$$

frequency shifts can also be determined for transitional water depth conditions from

$$\Delta f = f - f_0 = f_0\left(\frac{g}{g - \pi f_0 c(d)} - 2\right) \qquad (28)$$

provided that an estimate is made at first.

The solution is also contained in Fig. 4 in the form of a second family of curves with the parameter $f_0$ :

$$c = \frac{g f}{\pi f_0 (f + f_0)} \qquad (29)$$

Subscript o denotes once more wave parameters on deep water.
The diagram can be used not only for the determination of red shifts at decreasing water depth but also vice versa, and moreover between any two positions on a wave beam provided that the respective water depths are knwon.

Concerning the later case the frequency shift is :

$$\Delta\Delta f_{(d_1/d_2)} = \Delta f_{(d_o/d_2)} - \Delta f_{(d_o/d_1)} \tag{30}$$

and furthermore

$$\Delta\Delta f_{(d_1/d_2)} = f_o g \left[ \frac{1}{g-\pi\cdot f_o\cdot c_{(d_2,\ f_2)}} - \frac{1}{g-\pi\cdot f_o\cdot c_{(d_1,\ f_1)}} \right] \tag{31}$$

with $d_1 > d_2$ and $f_o$ = frequency on deep water.

The treatment of limiting values yields :

1. With the input frequency (initial frequency) $f_I = f_o$ approaching null, the frequency shift becomes also null and
2. with the input frequency $f_I = f_o$ approaching infinity the frequency shift gets infinite, too.

Input frequencies in between correspond to finite frequency shifts

at decreasing water depth, red shifts with a final frequency $f_f = 0$ and

at increasing water depth, blue shifts with a final frequency $f_f = \frac{g}{2\pi\ c_0}$ .

As the two families of curves in Fig. 4 do not form isogonal trajectories but intersect with angles $0^o < \gamma < 180^o$, the frequency shift varies with water depth. In general input frequencies, assigned by maximum frequency shifts, increase with the water depth decreasing.

For example, if there is a given representative wave period $T_1$ = 10 sec or frequency $f_1$ = 0.1 Hz respectively at a water depth $d_1$ = 20 m, an output frequency $f_2 = 0.067$ Hz can be determined at the water depth $d_2 = 10$ m by starting from the point [$f_1 = 0.1$ Hz ; $c(d_1 = 10$ m)] following the trend of wave beams (family of curves II) until the curve $c(d_2 = 10$ m) (of the set of curves I) will be reached.

At this range of water depths (20 m $\geq$ d $\geq$ 10 m) input frequencies $f_I = 0.05$ Hz < 0.1 Hz and $f_I = 0.2$ Hz > 0.1 Hz are assigned by smaller frequency shifts, whereas at a range of water depth 3 m $\geq$ d $\geq$ 2 m maximum frequency shifts belong to input frequencies $f_I \approx 0.25$ Hz and $f_I \approx 0.2$ Hz for wave propagation along with increasing or decreasing water depth respectively.

## 3. Conclusion

As is well known the dispersion relationship represented by the family of curves I in Fig. 4 describes a so-called normal dispersion. If, however, a frequency shift is considered an anomalous dispersion ($dc/df > 0$ or $dc/dL < 0$ respectively) turns out (family of curves II). Actually an anomalous dispersion is in accordance with the author's previous findings from analyses of measuring data (from the surf zone on the isle of SYLT/North Sea) in the time domain (with reference to a wave beam) as well as from evaluations in the frequency domain (with reference to the phase velocity components of deformed waves), see BÜSCHING (1978 a, 1978 b, 1979).

Contrary to the conservative treatment of theoretical wave transformation this leads to increasing wave length with the water depth decreasing and moreover all frequency components tend to approach frequency null, which corresponds to the state of rest.

Inserting $f = c/L$ into (29) delivers the family of curves II with reference to wave length L

$$c = \frac{g - \pi f_0^2 L}{\pi f_0} \qquad (32)$$

and furthermore (32) equated with (16) yields the corrected dispersion relationship

$$c = L \cdot f = \frac{f}{f_0} \left[ \frac{g}{\pi f_0} - \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}} \right] = \frac{f \cdot g}{\pi f_0 (f_0 + f)} \qquad (33)$$

As expected the respective limiting values of this equation are in accordance with that of (16) :

at deep water conditions, there is

$$c_0 = \frac{g\, T_0}{2\pi} = \frac{g}{2\pi f_0} \qquad (33\ a)$$

and at shallow water conditions, there is

$$c = \sqrt{gd} \qquad (33\ b)$$

With

$$\frac{dc}{dL} = - \frac{f}{2 f_0 L} \sqrt{\frac{gL}{2\pi} \tanh \frac{2\pi d}{L}} \left[ 1 - \frac{4\pi d}{L \cdot \sinh \frac{4\pi d}{L}} \right] \qquad (34)$$

the RAYLEIGH-relationship of group velocity is, however,

$$c_G = c - L \cdot \frac{dc}{dL} = c \left[ 1 + \frac{f}{2f_0} \left[ 1 - \frac{4\pi d}{L} \; \frac{1}{\sinh \frac{4\pi d}{L}} \right] \right] \qquad (35)$$

or shortened

$$c_G = n \cdot c \qquad (36\ a)$$

with

$$n = 1 + \frac{f}{2f_0} \left[ 1 - \frac{4\pi d}{L} \; \frac{1}{\sinh \frac{4\pi d}{L}} \right] \qquad (36\ b)$$

Provided that $f = f_o$, now it is on deep water

$$n = 1 + \frac{f}{2f_o} = \frac{3}{2} \qquad (37)$$

(as it is true to capillary waves), whereas in shallow water at the condition $dc/dL = 0$ we get

$$n = 1 \qquad (38)$$

In view of the fact that the present treatment is based on an energy transfer from one part of the spectrum into another, now it is questionable, whether the wave energy is still progressing with the group velocity $c_G$ or not.
If this were the case, the thesis of energy flux conservation could be applied in the form

$$H^2 \cdot L \cdot c_G = \text{konst.} \qquad (39)$$

yielding a relation with the wave heights by the following corrected shoaling coefficient :

$$K_{SK} = \frac{H}{H_o} = \frac{L_o}{L} \sqrt{\frac{3}{2n} \cdot \frac{f_o}{f}} = \frac{g}{2\pi f_o{}^2 L} \sqrt{\frac{3f_o}{2nf}} \qquad (40)$$

In the case of wave progress in line with decreasing water depth the result would be an increase of wave heights and lengths at the same time and thus an increasing wave energy in the coastward direction. In order to circumvent this kind of difficulties for the time being the thesis of energy conservation is directly applied on a wave beam of width 1 m with respect to linear wave theory

$$E = \frac{\rho g}{8} H^2 \cdot L = \text{konst.} \qquad (41)$$

Provided that energy losses need not be considered the change of wave heights with reference to deep water conditions is governed by

$$H(d) = H_o \sqrt{\frac{L_o}{L(d)}} = H_o \cdot T_o \sqrt{\frac{g}{2\pi c(d) \cdot T(d)}} = \frac{H_o}{f_o} \sqrt{\frac{g \cdot f(d)}{2\pi \cdot c(d)}} \qquad (42)$$

and in general the relationship between wave heights at any two positions on a wave beam is :

$$\frac{H_2}{H_1} = \sqrt{\frac{c_1 \cdot T_1}{c_2 \cdot T_2}} = \sqrt{\frac{c_1 \cdot f_2}{c_2 \cdot f_1}} \qquad (43)$$

Hence, at an anomalous dispersion along a wave beam progressing into shallower water the wave height decreases with the wave length and period increasing, whereas the contrary behaviour would turn out at wave progress into deeper water.

Thus in the case of finite water depth the theoretical wave transformation can be based completely on the data of Fig. 4.

In reality, however, the appearance of a distinct anomalous dispersion depends on the coincident presence of some important additional effects.
In particular the respective phenomena can obviously be amplified or compensated locally by superimposed accelerated currents ; it clearly turns out from equation (7) that velocity differences need only be small to produce a considerable frequency shift.

For example in the lower part of Fig. 6 there are shown some spectra which are measured synchronously at positions 100 m and 85 m from the shore.
If the respective sets of spectra are connected with the coast normal component of the mean residual velocities measured at Pos. 85 m (see middle part of Fig. 6) it can actually be seen that the frequency shift is modulated.
In the present case it seems that the frequency shift is

a. amplified in correspondence with a seaward directed residual current at Pos. 85 m (cf. first set of spectra)

b. affected minimal in correspondence with minimal residual currents (cf. second set of spectra) and

c. overcompensated by a coastward directed residual current (cf. third set of spectra).

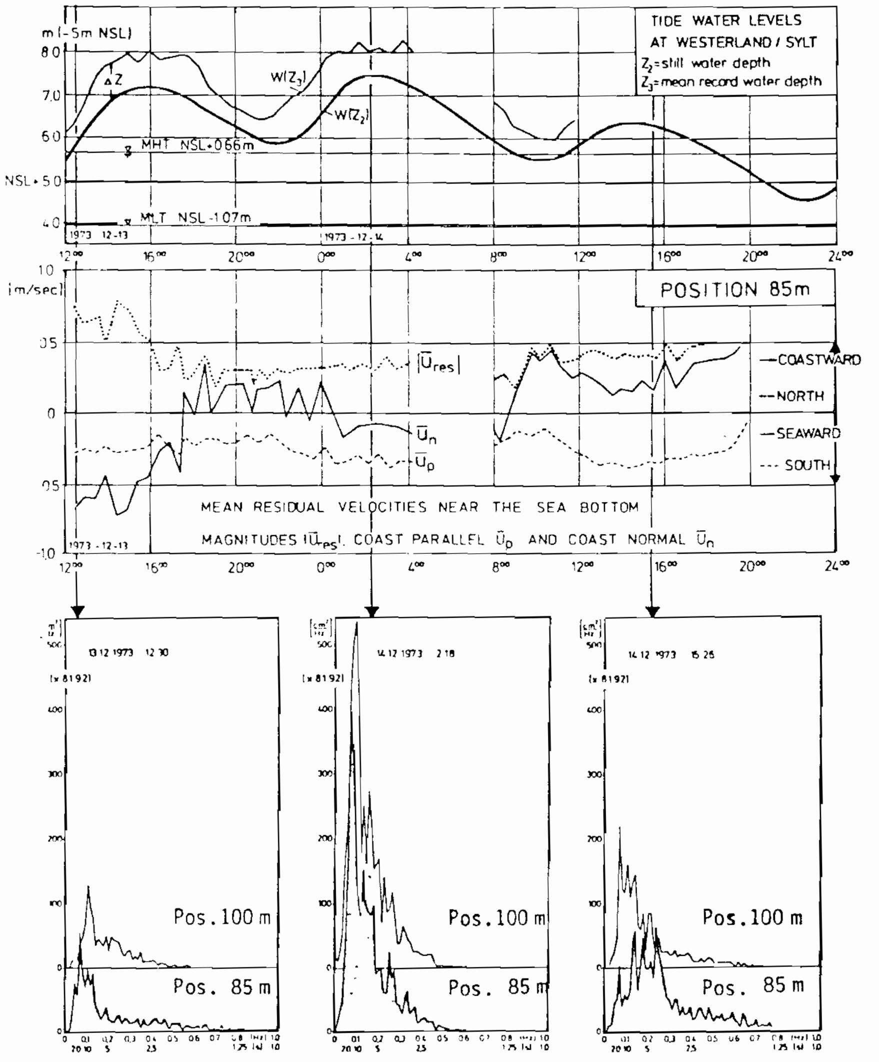


Fig. 6 : Modulation of the frequency shift by superimposed accelerated residual currents with respect to the onshore-offshore direction

As, however, a synchronous measurement of velocities at Pos. 100 m unfortunately is not available at the time being a final conclusion can not be drawn in this respect.
Furthermore the influence of bottom friction is responsible for significant changes at decreasing relative water depth d/L especially right next seaward of the wave breaking position.
Those effects will be studied in detail in future investigations.

## 4. References


ADEYMO, M. D. : Effect of Beach Slope and Shoaling on Wave Asymmetry. Proc. Eleventh Conf. Coastal Eng., London 1968.

BARBER, N. F., URSELL, F. : The generation and propagation of ocean waves and swell. 1. Philos. Trans. r. Soc. (A) 240, 257; 1948.

BIESEL, F. : Study of Wave Propagation in Water of Gradually Varying Depth. Gravity Waves Circular No.521, Nat. Bureau of Standards, Washington, D.C., 1951.

BÜSCHING, F. : Über Orbitalgeschwindigkeiten irregulärer Brandungswellen. Mitt. des Leichtweiß-Inst. der Techn. Universität Braunschweig, H. 42, 1974.

BÜSCHING, F. : On Energy Spectra of Irregular Surf Waves, Proc., 15th Int. Conf. Coastal Eng. Honolulu, 1976.

BÜSCHING, F. : Anomalous Dispersion of Fourier Components of Surface Gravity Waves in the Near Shore Area. Proc. 16th Int. Conf. Coastal Eng. Hamburg, 1978 (a).

BÜSCHING, F. : Wave Deformation due to Decreasing Water Depth. Mitt. des Leichtweiß-Inst. der Techn. Universität Braunschweig, H. 63, 1978 (b).

BÜSCHING, F.: Anomale Dispersion zur Darstellung der küstennahen Wellenverformung. Die KÜSTE, H. 34, 1979, Westholsteinische Verlagsanstalt Boyens u.Co., Heide i. Holst.

FÜHRBÖTER, A.: Einige Ergebnisse aus Naturuntersuchungen in Brandungszonen. Mitt. des Leichtweiß-Inst. der Techn. Univ. Braunschweig, H.40, 1974.

GALVIN, C. J.: Wave Breaking in Shallow Water in "Waves on Beaches". Edited by R.E. Meyer, Academic Press, Inc. New York and London, 1972.

HUBBLE, E. P.: A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae, Proc. National Academy of Sciences, 1929.

IWAGAKI, Y.
SAKAI, T.
KAWASHIMA, I.: On the Vertical Distribution of Water Particle Velocity by Waves on Beach. Coastal Engineering in Japan, 1973.

MADSEN, O. S.
MEI, C. C.: The Transformation of a Solitary Wave Over an Uneven Bottom. J. Fluid Mech. 39, 1969.

MULTER, R. H.
GALVIN, R. L.: Periodic Waves of Non-permanent Form (abstract) Trans. Am.Geophys.Union, 48, 1967.

PATRICK, D. A.
WIEGEL, R. L.: Amphibian Tractors in the Surf. Proc. First Conf. on Ships and Waves, The Engineering Foundation Council on Wave Research and the American Society of Naval Architects and Marine Engineers, 1955.

SONU, C. J.
PETTINGREW, N.
FREDERICKS, R. G.: Measurement of Swash Profile and Orbital Motion on the Beach. Proc. Intern. Symp. Ocean Wave Measurement and Analyses, New Orleans, 1974.

ZABUSKY, N. J.
GALVIN, C. J.: Shallow Water Waves. The Korteweg - De Vries Equations and Solitions, J. Fluid Mech. 47, 1971.

## 5. Abstract

Steadily increasing wave periods in the upbeach direction (turning out from strip chart registrations) as well as a red shift (resulting from frequency domain evaluations) both are in accordance with a theoretical treatment of water wave transformation based on DOPPLER's principle.

As a consequence, with the water depth decreasing the transformation along a wave beam is associated with an anomalous dispersion relationship also containing the frequency as a parameter.

It can be expected that the respective findings can present a better basis for the estimation of the still persisting influences on wave transformation; in particular bottom friction.

## 6. Zusammenfassung

Auswertungen von synchron aufgezeichneten Wasserspiegelauslenkungen (Wellen), die im küstennahen Bereich vor der Insel SYLT vorgenommen wurden, haben sowohl im Zeitbereich als auch im Frequenzbereich u.a. das Ergebnis einer mit abnehmender Wassertiefe verbundenen Wellenperiodenzunahme gezeigt. Diese im Gegensatz zur üblichen Betrachtung der Wellenverformung stehende Erscheinung wird unter Anwendung des DOPPLER-Prinzips auf die Dispersionsrelation erster Ordnung als sogenannte "Rotverschiebung" erklärt.

Infolgedessen liegt bezüglich der theoretischen Wellentransformation entlang eines "Wellenstrahls" eine anomale Dispersion vor, die frequenzabhängig ist. Es wird erwartet, daß die vorliegende Untersuchung insbesondere eine Grundlage dafür sein kann, den Einfluß der Bodenreibung auf die Wellenbewegung von anderen Einflußgrößen zu trennen.