Complex Reflection Coefficients 
Applied to Steep Sloping Structures

Supplement to:
„Phase Jump due to Partial Reflection of Irregular Water Waves at Steep Slopes“, 
Third Int. Conference COASTLAB 10, 2010, Barcelona, Spain, Paper Nr. 67

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Scope of Presentation

- **Analogue:**
  Phase shift $\Delta \phi$ between incident and reflected waves marks cases of *positive and negative partial reflection* at coastal structures.

- **Theoretical derivation of the Complex reflection coefficient**
  \[ \Gamma = C_r e^{i\Delta \phi} \]

- **Presentation of Measurements:**
  Magnitudes and Phases, \text{Re}[\Gamma] and \text{Im}[\Gamma] parts of the Complex reflection coefficient (CRC).

- **Discuss influence on Types of Breakers**

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Coastlab 2012, Ghent
Punging breaker on quasi Smooth Slope

$$C_r = 0.33; \Delta \phi \approx 216^\circ$$

Collapsing breaker on Hollow Cubes

$$C_r = 0.20; \Delta \phi \approx 163^\circ$$

Tests on the stability of Hollow Cubes

Model scale: 1:5

Slope: 1:n = 1:3

Wave heights up to $H = 0.35m$

$$\Delta x \approx 0.15m$$
Monochromatic Waves: 
Total wave field = incident + reflected wave

\[ y(x, t) = A e^{i(\omega t - kx)} + C_r A e^{i(\omega t + kx + \Delta \phi)} \]
\[ = (e^{-ikx} + C_r e^{i\Delta \phi} e^{ikx}) A e^{i\omega t} = (e^{-ikx} + \Gamma e^{ikx}) A e^{i\omega t} \]

Complex reflection coefficient \( \Gamma = C_r e^{i\Delta \phi} \)

considering magnitude \( C_r = A_r/A_i = H_r/H_i \) and phase shift \( \Delta \phi \)

Applying Euler’s formula, two special cases:

**Positive Total Reflection** where \( \Delta \phi = 0^\circ \) and \( C_r = 1 \) \( \rightarrow \) \( \Gamma = 1 \)

\[ y(x, t) = (e^{ikx} + e^{-ikx}) A e^{i\omega t} = 2 A \cos kx e^{i\omega t} \]
\( \rightarrow \) perfect standing wave **without** phase jump (Clapotis)

**Negative Total Reflection** where \( \Delta \phi = 180^\circ \) and \( C_r = 1 \) \( \rightarrow \) \( \Gamma = -1 \)

\[ y(x, t) = (e^{-ikx} - e^{ikx}) A e^{i\omega t} = -2 i A \sin kx e^{i\omega t} \]
\( \rightarrow \) perfect standing wave **with** phase jump \( \Delta \phi = 180^\circ \) (Clapotis)
Real part of the *komplex* reflection coefficient

\[ \Gamma = C_r e^{i \Delta \varphi} \]

**positive reflection**

**negative reflection**
Hollow Cubes piled up to form a stepped face hollow seawall structure (2-layer-system).
Slope: 1:2
Model scale: 1:10

Hollow Cubes $C_r \approx 0.2; \Delta \phi \approx -20^\circ$

Smooth slope $C_r \approx 0.7; \Delta \phi \approx 154^\circ$

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Energy Density Spectra of Water Level Deflections

(Composite Power Spectra)

Permeable revetment

(measured synchronously at slopes 1:3, distorted by re-reflection)

Plane revetment

256 Components

$\Delta f = 0.00543 \text{ Hz}$
Energy in front of slopes 1:2

Comments:

- Integrated spectrum area values ($E$) plotted with distance from IP: $\rightarrow$ partial Clapotis $L \approx 3.8m$

- Total Energy distributed to subfrequency ranges mark 10 component partial clapotis waves. $E_{\text{max}} = \text{loop}, E_{\text{min}} = \text{node}$

- The higher the frequency of partial clapotis waves the more they are shifted in upslope direction.

Close to IP
- nodes at smooth slope and
- loops at hollow slope.

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General properties of partial standing waves at a slope

Breakers

Partial Clapotis Envelopes

CRC Magnitude

$C_{r,i} = \frac{\sqrt{E_{\text{max},i}} - \sqrt{E_{\text{min},i}}}{\sqrt{E_{\text{max},i}} + \sqrt{E_{\text{min},i}}}$

where

$E_{\text{max},i}$ = maximum energy at loop $i$ and

$E_{\text{min},i}$ = minimum energy at node $i$

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Reflection Coefficient Magnitudes $C_r$ at slopes 1:2

Core of Spectrum

Smooth slope

Hollow slope

5 component partial standing waves representing the core of the spectrum.
General properties of partial standing waves at a slope

**Breakers**

- Loop I
- Node I
- H_{\text{max} I}
- H_{\text{min} I}
- \eta_{\text{max}}
- \eta_{\text{min}}

**Partial Clapotis Envelopes**

- Loop II
- Node II
- H_{\text{max} II}
- H_{\text{min} II}
- Loop III
- Node III
- H_{\text{max} III}
- H_{\text{min} III}
- Loop IV
- Node IV
- H_{\text{max} IV}
- H_{\text{min} IV}

**Partial Clapotis Energy**

- Loop nearest to IP: \( \Delta \varphi[^\circ] = 360(1 - 2\eta_{\text{max}}/L) \)
- Node nearest to IP: \( \Delta \varphi[^\circ] = 180(1 - 4\eta_{\text{min}}/L) \)

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Phase shifts $\Delta \phi$ of component partial standing waves

**Loop:**

$\Delta \phi = 360 \left(1 - 2\eta_{max}/L\right)$

$= 132.6^\circ$

**Horizontal wave assymmetry:**

$\eta_{max} - \eta_{min} \approx L/4$

**Node:**

$\Delta \phi = 180 \left(1 - 4\eta_{min}/L\right)$

$= 123.2^\circ$

$L \approx 2 \cdot 1.9m = 3.8m$

$\eta_{max} = 1.2m$

$\eta_{min} = 0.3m$

$\approx L/2$

$> L/4$

$< L/4$
Phase shifts $\Delta \phi \, [^\circ]$ at slopes 1:2

Loop:

$\Delta \phi = 360\pi (1 - 2\eta_{\text{max}}/L)$

$\Delta \phi = 132.6$
Complex Reflection Coefficients $\Gamma = C_re^{i\Delta\phi}$ at slopes 1:2

Gaussian Phasor Diagram

<table>
<thead>
<tr>
<th>f[Hz]</th>
<th>L[m]</th>
<th>$\eta_{max}$[m]</th>
<th>$C_r$</th>
<th>$\Delta\phi$[°]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Smooth Slope</td>
</tr>
<tr>
<td>0.475</td>
<td>4.4</td>
<td>1.40</td>
<td>0.83</td>
<td>130.9</td>
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<tr>
<td>0.503</td>
<td>3.8</td>
<td>1.20</td>
<td>0.80</td>
<td>132.6</td>
</tr>
<tr>
<td>0.544</td>
<td>3.4</td>
<td>1.00</td>
<td>0.68</td>
<td>148.2</td>
</tr>
<tr>
<td>0.578</td>
<td>3.0</td>
<td>0.90</td>
<td>0.69</td>
<td>144.0</td>
</tr>
<tr>
<td>0.594</td>
<td>2.9</td>
<td>0.80</td>
<td>0.70</td>
<td>161.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Hollow Cubes</td>
</tr>
<tr>
<td>0.475</td>
<td>4.4</td>
<td>2.15</td>
<td>0.09</td>
<td>8.2</td>
</tr>
<tr>
<td>0.503</td>
<td>3.8</td>
<td>1.95</td>
<td>0.16</td>
<td>-9.5</td>
</tr>
<tr>
<td>0.544</td>
<td>3.4</td>
<td>1.80</td>
<td>0.18</td>
<td>-21.2</td>
</tr>
<tr>
<td>0.578</td>
<td>3.0</td>
<td>1.65</td>
<td>0.21</td>
<td>-36.0</td>
</tr>
<tr>
<td>0.594</td>
<td>2.9</td>
<td>1.60</td>
<td>0.25</td>
<td>-37.2</td>
</tr>
</tbody>
</table>

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Smooth Slope

$1:n = 1:2$

- Magnitude $C_r$
- Relat. Loop Distance $\eta_{\text{max}}/L$

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Magnitudes $C_r$ and relative loop distances $\eta_{\text{max}}/L$

Hollow Slope

1:n = 1:2

- Magnitude $C_r$
- Relat. Loop Distance $\eta_{\text{max}}/L$
Real and imaginary parts of CRC $\Gamma = C_r e^{i\Delta \phi}$

Smooth Slope

$1:n = 1:2$

- **Re[$\Gamma$]**
- **Im[$\Gamma$]**

Frequency [Hz]

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Real and imaginary parts of CRC  \( \Gamma = C_r e^{i\Delta \varphi} \)

Hollow Slope

1:n = 1:2

- **Re[\(\Gamma\)]**
- **Im[\(\Gamma\)]**

Frequency [Hz]

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Average Complex Reflection Coefficients $\Gamma = C_r e^{i\Delta \phi}$

Different revetments on slopes 1:3 and 1:2

<table>
<thead>
<tr>
<th>$f$ [Hz]</th>
<th>$L$ [m]</th>
<th>$\eta_{\text{max}}$ [m]</th>
<th>$C_r$</th>
<th>$\Delta \phi$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth slope, 1:3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.38</td>
<td>3.65</td>
<td>0.73</td>
<td>0.33</td>
<td>216.0</td>
</tr>
<tr>
<td>to Hollow revetment, 1:3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.76</td>
<td>3.65</td>
<td>1.00</td>
<td>0.20</td>
<td>162.7</td>
</tr>
<tr>
<td>Smooth slope, 1:2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.36</td>
<td>3.5</td>
<td>1.00</td>
<td>0.72</td>
<td>154.3</td>
</tr>
<tr>
<td>to Hollow Cubes, 1:2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>3.5</td>
<td>1.85</td>
<td>0.20</td>
<td>-20.6</td>
</tr>
</tbody>
</table>

Gaussian Phasor Diagram

- Negative Total Reflection
- Positive Total Reflection
- $90^\circ$ (-270°)
- $270^\circ$ (-90°)
- $\pm 180^\circ$
Monochromatic waves at smooth slopes (1995): Relative node distances $\eta_{\text{min}}/L$ with slope angle and frequency.

\[
\tan \alpha = 1:m \quad 1:0.1 \quad 1:0.5 \quad 1:1 \quad 1:2 \quad 1:3
\]

$\eta_{\text{min}}/L$ to be converted to $\Delta \varphi[°]=180(1-4\eta_{\text{min}}/L)$

\[1.17 \leq T \leq 2.22 \text{s}\]
Results of monochromatic waves:

With respect to both axis (frequency and slope angle) there are

- **opposite trends of magnitudes and phase angles**,
- Trend decreasing for longer waves and
- Trend increasing for flatter slopes.
Types of breakers correlating to CRC?

Phase angle $\Delta \phi$ controls the positioning of the partial standing wave at the structure and consequently the location of the breaker depth.

**Conclusion:** Phase angle $\Delta \phi$ is **needed** for a complete description of the breaking at a slope.

**Presumptions** on types of breakers correlating to phase angles:

<table>
<thead>
<tr>
<th>Phase angle $\Delta \phi$</th>
<th>Type of breaker</th>
<th>Further condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\approx 0^\circ$</td>
<td>broken Clapotis</td>
<td>super critical steepness</td>
</tr>
<tr>
<td>1st or 4th quadrant</td>
<td>no distinct breaker type</td>
<td>dissipation &gt; transmission</td>
</tr>
<tr>
<td>(positive reflection)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\approx 180^\circ$</td>
<td>surging breaker</td>
<td>low dissipation</td>
</tr>
<tr>
<td>2nd or 3rd quadrant</td>
<td>collapsing or plunging breaker</td>
<td>dissipation and transmission</td>
</tr>
<tr>
<td>(negative reflection)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Measurements too few for more consistent statements!
Trivial suggestions on the **joint effect of reflection, transmission and dissipation** of breaking waves at a slope

Verify current findings in a **natural** scale:
Specify phase shifts for **longer waves** and **less inclined** slopes.

Consider the phase difference $\Delta \varphi$ in the presentations of both
- the reflection coefficients and
- the types of breakers
as functions of the **Iribarren** Number

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